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HAAR WAVELET FOR NUMERIC SOLUTION OF RLC CIRCUIT DIFFERENTIAL EQUATIONS

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ABSTRACT

The wavelet transformation is a mathematical method developed over the past decades to be adapted for applications in the fields of science and engineering. The wavelet transform can be applied in the field of numerical analysis to solve the differential equation. This paper is concerned with applying Haar wavelet methods to solve an ordinary differential equation for an RLC series circuit with a known initial state. The matrix construction calculations are proposed in a simple way. Three numerical mathematical examples are shown that include second-order differential equations with variable and constant coefficients. The results showed that the proposed method is quite reasonable while comparing the solution of second order systems by Haar wavelet method with the exact solution in the context of serial RLC circuit. Moreover, the use of Haar waves is found to be simple, accurate, with flexible and appropriate arithmetic computational costs.

KEYWORDS: Haar Wavelets, Ordinary Differential Equations, Matrix Representation, RLC series Circuit.

1. INTRODUCTION

Wavelet analysis is recently used as mathematical tools for the benefits of science and many engineering field. Recently, the wavelet transform has been applied extensively in various aspects such as denoising data, data compression, image compression, and many more (Galli et al., 1996).

Wavelet analysis includes large computations, almost; the calculation is achieved by using software application with special toolboxes and commands, it may make a ruthless impression for beginners (Sadiku et al., 2005). In time being, Haar function has been select for educational purpose, essentially in many books or papers written on subject of introduction to wavelets (Nason, 1999; Aboufadel and Schlicker, 1999; Mustafa, 2021), due to the robust of wavelets in important fields of science and engineering.

In this paper, the operator or matrix representation is developed in a wavelet basis. Sometime, the tasks for writing up the operational matrices are too long essentially, when someone intends to perform the computation in high resolution. This will discourage the beginner to know how wavelet basis can be done to solve differential equations, principally when in the works of encouraging the research of wavelets in undergraduate field which were discussed and achieved in (O'neil, 2003; Potter et al., 2005).

This paper is organized as follows. In section 2 the literature review for recent work is discussed. In section 3 Haar wavelet and their integrals is described and in section 4 haar matrices are established. Expanding functions into the Haar Wavelet series is discussed in section 5. Approximating solutions of differential equation is given in section 6. Exact solution of Series RLC circuit is presented in section 7, numeric solution results of series RLC circuit using Haar wavelet

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are reported in section 8. Finally, the conclusion is specified in section 9.

2. LITERATURE REVIEW

Wavelet transform was introduced in the area of numerical analysis set in the early 1990s (Schneider and Vasilyev, 2010) and based procedure have become a significant tools because of the characteristics of localization. Haar wavelet is one of the common families of wavelet. Because of its simplicity, Haar wavelets had become an efficient tool for solving many problems, among that are Partial Differential Equations, PDEs and Ordinary Differential Equations, ODEs (Lepik, U. 2008).

Haar wavelets have been used in different numerical approximation problems due to their orthonormal properties with compact support; i.e. it is nonzero only on a finite interval. Chen and Hsiao (1997; 1999) who first concluded an operational matrix of integration based on the Haar wavelet transform. Stankovic and Falkowski (2003) achieved different generalizations of the Haar transform as Haar functions, sign Haar transform, Zhang-Moraga functions, p-adic groups. Lepik and Tamme (2004) have proposed the Haar wavelets for solving linear and non-linear integral equations. Maleknejad and Mirzaee (2005) have used linear integral equation based Haar wavelets. Lepik (2008) used Haar wavelet method in solving higher order as well as nonlinear ODEs.

Based on Haar wavelet method, Prabakaran et al. (2014) used Haar wavelet series method to get discrete solutions for a state space system of differential equations. The author Arbabi et al. (2017) proposed an efficient algorithm to solve systems of three-dimensional nonlinear partial differential equations. The algorithm is based on Haar wavelet method and a quasi-linearization technique. The work of Sahar et al. (2019) use Haar wavelet numerical scheme to derive the solutions of the fractional electrical circuits, namely RC, LC and RLC. Haar wavelet collocation method for solving the telegraph equation is given by the author Nagaveni K (2020) to obtaining the approximate solution for two different types of telegraph equations. The authors' Waleeda et al. (2022) proposed a Haar wavelets method utilized to approximate a numerical solution for linear state space systems. Applications of Haar wavelet for numerical approximations are indicated by the authors' Imran et al. (2022), explores numerical solutions to elliptic, parabolic and hyperbolic equations with two different types of nonlocal integral boundary conditions. The numerical solutions are obtained using the Haar wavelet collocation method with the aid of Finite Differences for time derivatives. Recently, Haar wavelet method has been extended to solve fractional partial differential equations as can be found in research article by Rohul et al. (2022), they developed a computational Haar collocation scheme for the solution of fractional linear integro- differential equations of variable order.

Motivated by the excellent performance of the above methods, in this work the same techniques are used with lower functional evaluation of numerical analysis. The Haar wavelet for numeric solution of differential equations has been applied to solve the second-order differential equation, and the output voltage in a serial RLC circuit is numerically determined.

3. HAAR WAVELET AND THEIR INTEGRALS

Haar wavelets have been used as an earliest example for orthonormal wavelet transform or wavelet analysis with strong support. The Haar wavelet is the first known wavelet and was achieved in 1909 by Alfred Haar. By considering the interval $t \in [0,1)$, the father wavelet or Haar scaling function $h_1(t)$ that appears in the form of a square wave as:

$$h_1(t) = \begin{cases} 1 & for\ 0 \le t < 1, \\ 0 & elsewhere. \end{cases} \quad (1)$$

Correspondingly, there is a mother wavelet function or the Haar wavelet $h_2(t)$ which is described in figure 1 and as follows:

$$h_2(t) = \begin{cases} 1 & for\ 0 \le t < \frac{1}{2}, \\ -1 & for\ \frac{1}{2} \le t < 1, \\ 0 & elsewhere. \end{cases} \quad (2)$$

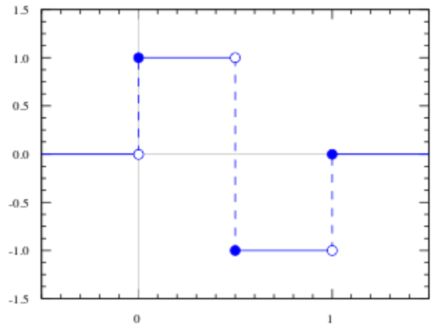


Fig.(1):- Haar wavelet $h_2(t)$

The other levels of wavelets can all be generated from $h_2(t)$ by the combined operations of translation and dilations. The general formula for the family of Haar wavelets up to maximal level J of decomposition can be written for t ∈ [0,1) as:

$$h_i(t) = h_2(2^j t - k) = \begin{cases} 1 & for\ \ t \in [X_1, X_2), \\ -1 & for\ \ t \in [X_2, X_3), \\ 0 & elsewhere. \end{cases} \quad (3)$$

Where $X_1 = k2^{-j}$, $X_2 = (k+0.5)2^{-j}$ and $X_3 = (k+1)2^{-j}$

Follow the work done by (Lepik, 2005), the dilatation parameter is defined as $m = 2^j$, for j = 0, 1, . . . ,J (J the maximal level of resolution) and the translation parameter are k = 0, 1, . . . , m − 1. The wavelet number i is recognized as $= m + k + 1$; in the case of minimal values m = 1, k = 0 we have i = 2. It is assume that the value i = 1 corresponding to the scaling function for which $h_1(t) = 1$ for t ∈ [0 1). The quantity $M = 2^J$ is defined in term of J the maximal level of resolution. Therefore, the maximum value of $i = 2M = 2^{J+1}$. if the interval $[0,1)$ is divided into $2M$ subintervals of equal length; the length of each subinterval is $\Delta t = 1/(2M)$. The association grid points t_l are defined as:

$$t_l = (l - 0.5)\,\Delta t = \frac{2l-1}{4M} \quad for\ l = 1, 2, \cdots, 2M \quad (4)$$

These collocation points t_l represent discrete samples determined from the Haar function $h_i(t)$. The orthogonal set of haar wavelets $h_i(t)$ for $i = 1, 2, \cdots, 8$ are given in Table 1 at two level of ecomposition (J=2, M=4), the subintervals of equal length is $\Delta t = \frac{1}{2M} = \frac{1}{8} = 0.125$. From (4) there are eight collocation points using the formula $t_l = (2l-1)/16\ for\ l = 1, 2, \cdots, 8$; as follows: t_1=1/16, t_2=3/16, t_3=5/16, t_4=7/16, t_5=9/16, t_6=11/16, t_7=13/16, and t_8=15/16.

Table (1) :-The orthogonal set of Haar wavelets $h_i(x)$ at two level of decomposition (J=2, M=4), the subintervals of equal length

$\Delta x = \frac{1}{2M} = \frac{1}{8} = 0.125$ is eight and $t_l = \frac{2l-1}{16}$ $for\ l = 1, 2, \cdots, 8$

j	m	k	i	$h_i(x)$	t_1 0.0625	t_2 0.1875	t_3 0.3125	t_4 0.4375	t_5 0.5625	t_6 0.6875	t_7 0.8125	t_8 0.9375
0	1	0	1	$h_1(x)$	1	1	1	1	1	1	1	1
0	1	0	2	$h_2(x)$	1	1	1	1	–1	–1	–1	–1
1	2	0	3	$h_3(x) = h_2(2x)$	1	1	–1	–1	0	0	0	0
1	2	1	4	$h_4(x) = h_2(2x-1)$	0	0	0	0	1	1	–1	–1
2	4	0	5	$h_5(x) = h_2(4x)$	1	–1	0	0	0	0	0	0
2	4	1	6	$h_6(x) = h_2(4x-1)$	0	0	1	–1	0	0	0	0
2	4	2	7	$h_7(x) = h_2(4x-2)$	0	0	0	0	1	–1	0	0
2	4	3	8	$h_8(x) = h_2(4x-3)$	0	0	0	0	0	0	1	–1

To solve a n-th order ODE, the integration function is required. In (Chen and Hsiao, 1997) the given method is:

$$p_{1,i}(t) = \int_0^t h_i(u)du \tag{5}$$

Then,

$$p_{1,i}(t) = \begin{cases} t - X_1 & for\ \ t \in [\,X_1, X_2\,), \\ X_3 - t & for\ \ t \in [X_2, X_3\,), \\ 0 & elsewhere. \end{cases} \tag{6}$$

However, for v-times integration

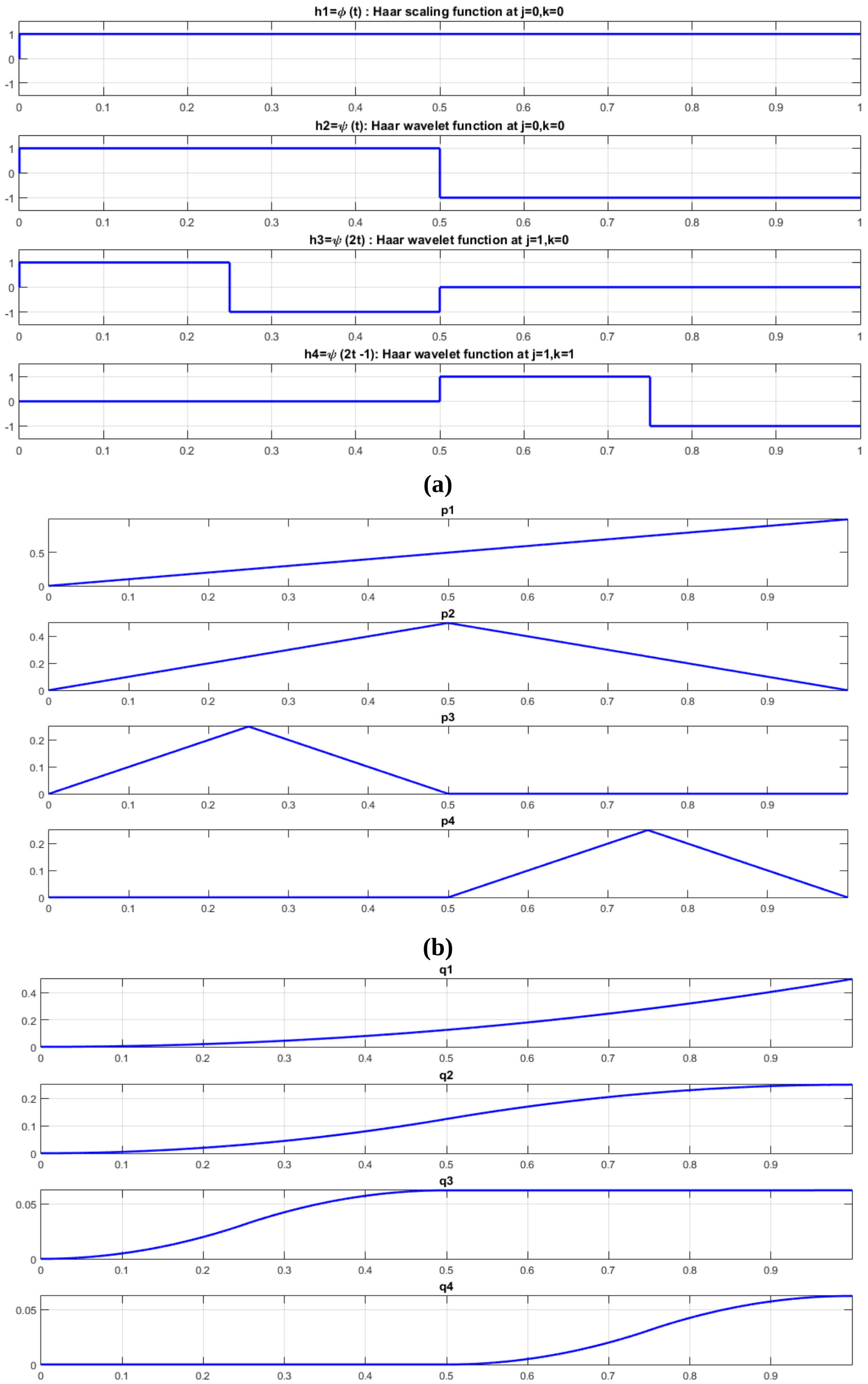
$$P_{v,i}(t) = \int_0^t P_{v-1,i}(u)du,\ \ for\ v = 2, 3, \ldots, n \tag{7}$$

For $v = 2$, the integration is:

$$p_{2,i}(t) = \begin{cases} \frac{1}{2}(t - X_1)^2 & for\ \ t \in [\,X_1, X_2\,), \\ \frac{1}{4m^2} - \frac{1}{2}(X_3 - t)^2 & for\ \ t \in [\,X_2, X_3\,), \\ \frac{1}{4m^2} & for\ \ t \in [\,X_3, 1\,), \\ 0 & elsewhere \end{cases} \tag{8}$$

Carrying out these integrations in the same manner, the other values of $P_{v,i}(t)$ can be computed.

The resulting $p_{1,i}(t)$ is a triangular shape, whereas $p_{2,i}(t)$ is the parabolic one. The graph of first four Haar wavelets of $h_i(t)$ at two level of decomposition (J=2) and their corresponding integrals, $p_{1,i}(t)$ and $p_{2,i}(t)$ is shown in figure 2.



(c)

Fig.(2): -Haar wavelets: (a) $h_i(t)$ at two level of decomposition (J=2) and their corresponding integrals, (b) $p_{1,i}(t)$ and (c) $p_{2,i}(t)$.

4. HAAR MATRICES

The group of orthogonal set of Haar wavelets $h_i(x)$ at J level of decomposition has $2M = 2^{J+1}$ Haar functions $h_i(t)$, and each Haar function has $2M$ sample points. Therefore, a Haar matrix H_N is defined with dimension $\mathrm{N} \times \mathrm{N}$, where $\mathrm{N} = 2\mathrm{M}$. For example if $\mathrm{J} = 1, \mathrm{N} = 4$. By calculating the coordinates of the collocation points from (4) we find t_1= 0.125, t_2 = 0.375, t_3 =0.625, t_4 = 0.875. The Haar matrix is:

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$$H_4 \triangleq \begin{bmatrix} h_1(x) \\ h_2(x) \\ h_3(x) \\ h_4(x) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

The operation matrix P_1 is defined to represents the first order integration $p_{1,4}(t)$, for Haar matrix H_4. While operation matrix P_2 represents the second order integration $p_{2,4}(t)$ for Haar matrix H_4. Therefore, the matrices $P_{1,4}$, $P_{2,4}$ are:

$$P_{1,4} = \frac{1}{8}\begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 3 & 3 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad P_{2,4} = \frac{1}{128}\begin{bmatrix} 1 & 9 & 25 & 49 \\ 1 & 9 & 23 & 31 \\ 1 & 7 & 8 & 8 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

5. EXPANDING FUNCTIONS INTO THE HAAR WAVELET SERIES

Any function f(t) which is square integral in the interval [0, 1) can be expanded in a Haar wavelet series as:

$$f(t) = \sum_{i=1}^{2M} a_i h_i(t) = aH \qquad (9)$$

The symbol a_i denotes the Haar wavelet coefficients. The Haar wavelet coefficients are computed as inner product $a_i = \langle f(t), h_i(t)\rangle = 2^j \int_0^1 f(t)\, h_i(t) dt$, the discrete form of integral is $\Delta t \sum_{l=1}^{2M} f(t_l)\, h_i(t_l)$. Now Haar wavelet coefficients a_i can be computed as

$$a_i = 2^j \Delta t \sum_{l=1}^{2M} f(t_l)\, h_i(t_l) \qquad (10)$$

Here, H is the Haar matrix and a as coefficient matrix; both are 2M dimensional row vectors. By solving the matrix equation with regard to the coefficient vector *a*, then:

$$a = fH^{-1} \qquad (11)$$

(H^{-1} denotes the inverse of H).

Intended for the first four Haar functions matrix, suppose the $f = [4 \;\; 8 \;\; 1 \; 3]$ as piecewise constant then

$a = fH^{-1} = [4 \quad 2 \; -2 \; -1]$ and $f(t) = 4\, h_1(t) + 2\, h_2(t) - 2\, h_3(t) - h_4(t)$.

6. APPROXIMATING SOLUTIONS OF DIFFERENTIAL EQUATION

The first step to find the solution of n-th order linear ODE started in the form (Lepik, 2009):

$$y^{(n)}(t_l) = \sum_{i=1}^{2M} a_i h_i(t_l) = aH \qquad (12)$$

Lower order derivatives (and the function y(x)) are obtained through integrations of equation (12). All these ingredients are incorporated into equation (12) which is discretized by the collocation method. In this way, a system of equations is acquired from which the wavelet coefficients a_i are calculated.

In order to investigate the accuracy of such algorithms. Let exact solution of the problem is $y_{ex}(t_l)$. Then the mean square error is given by:

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$$mserror = \frac{1}{2M}\sum_{l=1}^{2M}(y(t_l) - y_{ex}(t_l))^2 \quad (13)$$

To demonstrate the approximation of solutions to deferential equations using Haar wavelet series, consider the general second order differential equation is:

$$y^{(2)}(t) + C_1 y^{(1)}(t) + C_2 y(t) = f(t)\ , where\ t \in [0, 1)$$

The boundary conditions $y^{(1)}(0) = y_1,\ \ y(0) = y_o$ are known.
To begin the approximation, let

$$y^{(2)}(t) \approx \sum_{i=1}^{2M} a_i h_i(t) = aH \quad (14)$$

Now integrate with respect of *t*

$$y^{(1)}(t) = \sum_{i=1}^{2M} a_i p_{1,i}(t) + y^{(1)}(0) = aP_1 + y_1 \quad (15)$$

Integrate $y^{(1)}(x)$ with respect of *t* to get the solution

$$y(t) = \sum_{i=1}^{2M} a_i p_{2,i}(t) + ty^{(1)}(0)\ + y(0) = aP_2 + ty_1 + y_o \quad (16)$$

Next, substitute these expressions into the general second order differential equation

$$aH + C_1(aP_1 + y_1) + C_o(aP_2 + ty_1 + y_o) = f(t)$$

and simplify to obtain

$$a(H + C_1P_1 + C_2P_2) = f(t) - (C_1y_1 +\ C_2ty_1 + C_2y_o) \quad (17)$$

In matrix form

$$aU = F - \text{B, where } U = (H + C_1P_1 + C_2P_2) \text{ and } B = C_1y_1 +\ C_2(ty_1 + y_o) \quad (18)$$

All quantities are calculated in the collocation points. Solve the matrix

$$a = U^{-1}(F -\ B)^T \quad (19)$$

The unknown wavelet coefficients vector *a* are obtained from solving equation after evaluating x at the collocation points. The solution of the *y* is found by substituting in equation (16) $y(t) = aP_2 + ty_1 + y_o$.

7. EXACT SOLUTION OF SERIES RLC CIRCUIT (SECOND DIFFERENTIAL EQUATION)

The discussion is developed to compare the solution of second order systems through the Haar wavelet approach with the exact solution in the context of the example of a series RLC circuit. The procedure of solving the second order differential equation using Haar wavelet are applied to find the output voltage $V_{out} = v_C$ on a capacitor through series RLC circuit with forced input voltage source $V_{in} = V_s$ as shown in figure 3. Analysis of such a circuit yields a second-order partial differential equation. The second order differential equation for the below circuit can be derived as (Berwal et al., 2013).

$$v_L + v_R + v_C = V_S \quad (20)$$

Where $v_L = L\frac{di}{dt}$ and $v_R = Ri$, the current in the circuit is $i = C\frac{dv_c}{dt}$. Therefor (18) can be written in term of v_C as:

$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = V_S(t) \qquad (21)$$

The complete solution of equation (21) is

$$v_C = v_{tr} + v_{ss} \qquad (22)$$

Where v_{tr} is the transient solution (homogenous), and v_{ss} is the steady-state solution (particular).

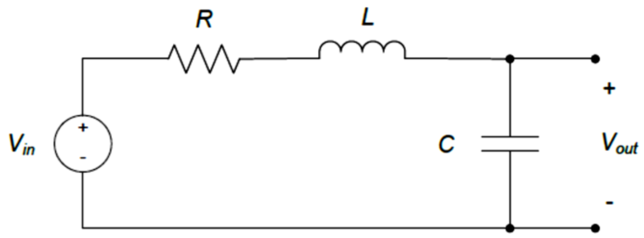

Fig.(3):- The series RLC circuit

For homogeneous solution set $v_{ss} = 0$ in equation (22). Depending on the value of the resistance connected to our energy storage elements, the transient response of an RLC circuit could be either over damped (decaying exponentially) or under damped (decaying, but oscillatory), with a "special case" of critically damped which is difficult to achieve in practice. In terms of the components of the series RLC circuit given in figure 3, the exponential damping factor is defined as $(a = \frac{R}{2L})$ and the resonant frequency is defined as $(\omega_o = \frac{1}{\sqrt{LC}})$. The initial conditions at t=0 are capacitor voltage $v_C(0)$, and it's

derivative $v'(0) = \left.\frac{dv}{dt}\right|_{t=0} = \frac{i_C(0)}{C} = 0$.

In this work, an input voltage source $V_S(t)$ is selected as step function. Thus, the forced function to the system can be written as:

$$V_S(t) = V_f\, u(t) = \begin{cases} 0 & if\ t < 0 \\ V_f & if\ t > 0 \end{cases} \qquad (23)$$

Where $u(t)$ is unit step function that makes sudden change at t = 0 from 0 to constant voltage V_f .The response of a system to this type of input is called the step response of the system. The particular solution of the differential equation can be gained by examining the solution to the equation after the homogeneous solution has died out. Letting t $\rightarrow \infty$ in equation (21) and noting that the strengthens function is a constant V_f as t $\rightarrow \infty$ allows us to set $v_L(t \rightarrow \infty) = v_R(t \rightarrow \infty) = 0$, and thus

$$v_{ss} = v_c(t \rightarrow \infty) = V_f \qquad (24)$$

Combining the particular and homogeneous solutions, the complete solution of $v_C(t)$ in equation (22) for series RLC circuits, after employing the initial conditions results in the complete solution for the step response of a second order system are summarized in table 2.

The L and C component values of circuit of figure 3 are assumed as L=1 Henry and C= 10^{-3}Farad. The value of R will be chosen to satisfy the above cases. Select the value of R to find the complete solution of v_c according to given values of L and C. The Complete exact solution of output $v_c(t)$ for three cases is given in table 3 for $V_f = 12$ volt. The resonant frequency $\omega_o = 31.4159\, rad/sec$ with initial conditions $v_c(0) = 1$ and $v_c'(0) = 0$ .The value of R is selected in order to satisfy each case. The Complete exact solutions of the three cases are specified in table 3 according to selected value of R.

Table (2):- Summary of equations for series RLC circuits with initial conditions $v_c(0) = 1$ and $v_c'(0) = 0$ with sudden application of a constant voltage V_f at $t = 0, V_S = V_f\ u(t)$.

Case	Condition	Criteria	Solution
1	Over damped	$a > \omega_o$	$v_c(t) = (v_c(0) - V_f)\left(\frac{\exp(s_1 t)}{1 - \frac{s_1}{s_2}} + \frac{\exp(s_2 t)}{1 - \frac{s_2}{s_1}}\right) + V_f$ $s_{1,2} = -a \pm \sqrt{a^2 - \omega_o{}^2}$
2	Critical damping	$a = \omega_o$	$v_c(t) = (v_c(0) - V_f)(1 + at)\exp(-at) + V_f$ $a = \frac{R}{2L} = \frac{1}{\sqrt{LC}}$
3	under damped	$a < \omega_o$	$v_c(t) = (v_c(0) - V_f)\exp(-at)\left(cos(\omega_d t) + \frac{a}{\omega_d}\sin(\omega_d t)\right) + V_f$ where $\omega_d = \sqrt{\omega_o^2 - a^2}$

Table(3):-Complete Solution of for three cases given $\omega_o = 31.4159\ rad/sec$ (L=1 Henry and C= 10^{-3}Farad). The initial conditions$v_c(0) = 1$, $v_c'(0) = 0$, and sudden application of a constant voltage $V_f = 12$ at $t = 0$.

Case	*R* Ohm	*a* sec^{-1}	Solution
1 Over damped $a > \omega_o$	125.6637	62.8319	$v_c(t) = -11\ (1.0786\exp(-8.5378t) - 0.0786\exp(-117.1259t)) + 12$
2 Critical damping $a = \omega_o$	62.8319	31.4159	$v_c(t) = -11(1 + 31.4159\ t)\exp(-31.4159t) + 12$
3 Under damped $a < \omega_o$	12.5 $\omega_d = 31$	6.25	$v_c(t) = -11\exp(-6.25t)\,(cos(31t) + 0.2016\sin(31t)) + 12$

8. NUMERIC SOLUTION RESULTS OF SERIES RLC CIRCUIT USING HAAR WAVELET

The numeric solution results for capacitor voltage $v_C(t_l)$ of series *RLC* circuit using Haar wavelet and the plot of numeric solution for all the three cases are shown in figures 4, 5 and 6 for all three cases the Over damped, Critical damping, and Under damped respectively. The obtained *mserror* which computed at each case is given in table 4.

Table(4):- The obtained *mserror* at each case at resolution J=5.

Case	$mserror = \frac{1}{2M}\sum_{l=1}^{2M}(v_C(t_l) - v_{C_{ex}}(t_l))^2, M = 2^J, J = 5$
Over damping	0.00012
Critical damping	0.00089
Under damping	0.0026

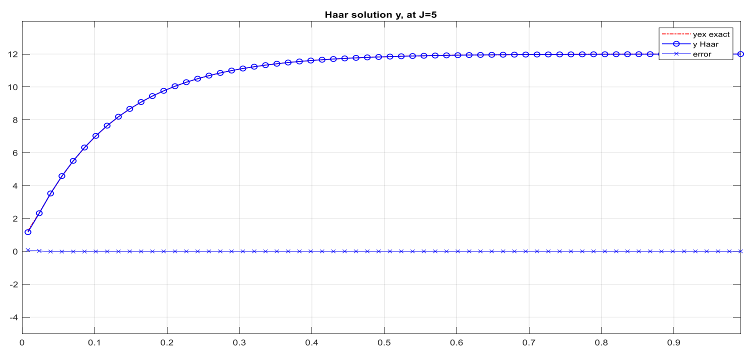

Fig.(4):- The upper plot represent the overdamped exact solution (red) and Numeric solution (blue) using Haar wavelet at resolution level J=5 of capacitor voltage $v_C(t_l)$ of the series RLC circuit. The lower plot is the instant difference between exact solution and Numeric solution.

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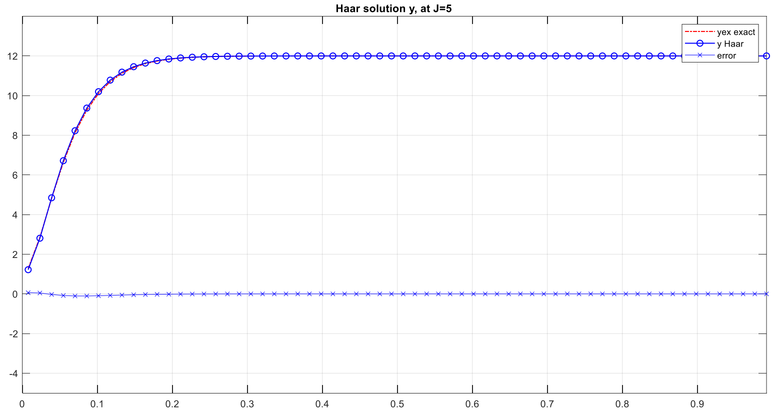

Fig.(5):-: The upper plot represent the critical exact solution (red) and Numeric solution (blue) using Haar wavelet at resolution level J=5 of capacitor voltage $\boldsymbol{v_C(t_l)}$ of the series RLC circuit. The lower plot is the instant difference between exact solution and Numeric solution.

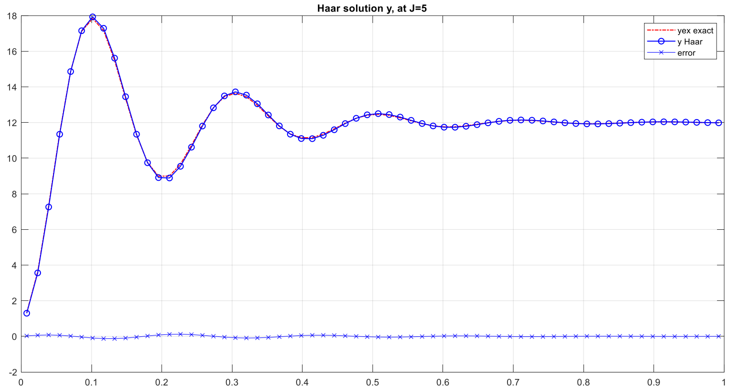

Fig.(6):- The upper plot represent the underdamped exact solution (red) and Numeric solution (blue) using Haar wavelet at resolution level J=5 of capacitor voltage $v_C(t_l)$ of the series RLC circuit. The lower plot is the instant difference between exact solution and Numeric solution.

The matlab codes were used in this research to find numeric solution of RLC circuit differential equations. As an evaluation performance, numerical calculations were carried out for different values of resolution level J of capacitor voltage $\boldsymbol{v_C(t_l)}$ for RLC overdamped using haar wavelet. Table 5 shows, the number of samples of output voltage $\boldsymbol{v_C(t_l)}$, Subinterval Δt , execution time required to execute matlab codes and the mean square error mserror of numeric solution along with resolution level J from 1 to 8. If the accuracy of these results is insufficient, more results that are precise could be obtained by increasing the J. However, it follows from table 5 that already in the cases J = 4 or higher the results visually coincide with the exact solution. The matlab codes off line execution time gave no significant influence because maximum time at J=8 is about 4 ms. The execute time and the mean square error of numeric solution using Haar wavelet at resolution level J = 4, 5 and 6 is shown in figure 7.

Table(5):- The evaluation performance, for different values of resolution level J of capacitor voltage $v_C(t_l)$ for RLC overdamped using haar wavelet

resolution level J	$2M = 2^{J+1}$ No. of samples of output voltage $v_C(t_l)$	$\Delta t = 1/(2M)$ Subinterval	Execution Time (μsec.)	mean square error $mserror$
1	4	0.25	10.0820	294.8774
2	8	0.125	11.9470	60.4314
3	16	0.0625	13.5170	0.2053
4	32	0.0313	31.2020	0.0042
5	64	0.0156	109.6900	12.567×10^{-5}
6	128	0.0078	315.8800	6.4535×10^{-6}
7	256	0.0039	958.4400	3.9585×10^{-7}
8	512	0.0020	3800	2.4712×10^{-8}

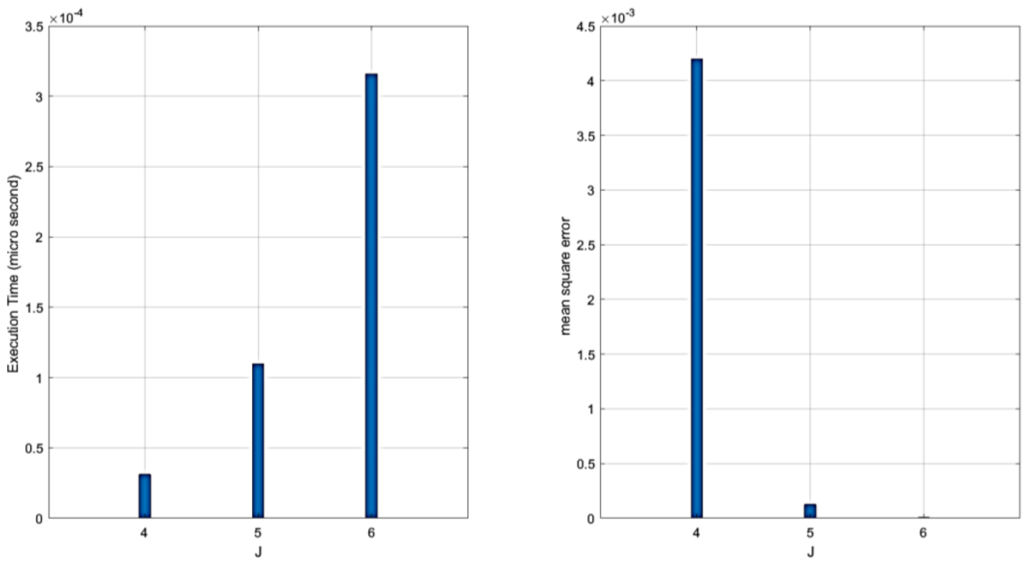

Fig.(7):- Execute time and the mean square error mserror of numeric solution using Haar wavelet at different resolution level J.

9. CONCLUSIONS

The procedure and algorithm that has been utilized by using the Haar wavelet method for solving ODEs show that the major advantage of this method is simplicity, which is due to the ability of the transform matrices and to the small number of significant wavelet coefficients. The results of this work show that the concept of using the Haar wavelet method as a powerful process for solving ordinary linear differential equations by taking the series RLC equations has been proved. The numeric solution is comparable to the over-damped, critical-damped, and under-damped exact solution. On observing the results, it can be agreed that the results obtained from the proposed Haar wavelet method for solving ODEs have good agreement with analytic solutions. The analysis shows that the proposed method is efficient and relevant for solving differential equations. The setting of proposed method for solving ODEs is easy and straightforward. The proposed method can be extended to solve two or higher order differential equations problems in diverse areas of physical, mathematical sciences and engineering applications.

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