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NUMERICAL SOLUTION OF HIROTA-SATSUMA COUPLED KDV SYSTEM BY RBF-PS METHOD

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ABSTRACT

In this paper, the Hirota-Satsuma coupled Korteweg-de Vries system is solved numerically by using radialbasis function-Pseudospectral method. The radial basis functions are used to approximate the space derivatives in the system. Moreover, the system has become a system of ordinary differential equations with independent variable , and this system is solved by Runge-Kutta fourth order method, with the help of MATLAB R2020a. Also, a comparison has been made between approximate solutions obtained by the proposed method and exact solutions.

KEY WORDS: Hirota-Satsuma Coupled KdV system; Radial basis functions; Pseudospectral method; Runge-Kutta 4th order method

1- INTRODUCTION

The nonlinear system of partial differential equations (PDEs) has variety of applications in different fields of mechanics, biology, hydro dynamics, and plasma physics. The Korteweg-de Vries (KdV) equations have been part of an important class of non-linear evolution equations with numerous applications in physics, plasma and engineering fields. In the theory of rogue waves, the KdV equation describes the effects in shallow water. In plasma physics, the KdV equations produce ion-acoustic solutions (Gao and Tian, 2021; Mahmoud et al., 2020). Also, the Hirota-Satsuma coupled KdV system has many applications in many branches of nonlinear science. For example, this equation can be applied to the field of thermodynamics, where it can be used to exactly calculate partition and correlation functions. Describing generic properties of string dynamics for strings and multi-strings in constant curvature space can be thought as another application of the Hirota-Satsuma coupled KdV system (Gao and Tian, 2021; Yucel et al., 2017).

There are many methods for approximating the solution of PDEs or the system of PDEs. Generally speaking, they fall into two classes: those in which the solution is approximated at some discrete points called grid or mesh points, and those in which the solution is approximated by a finite number of terms of infinite expansions concerning a sequence of functions (Ferreira et al., 2009; Jain, 1979).

In this paper, we used we use radial basis function (RBF) to approximate the space derivatives, because RBFs are increasingly being applied in the numerical solution of PDEs, and are a viable alternative to more traditional methods (Buhmann, 2004). The simple idea of using RBFs to solve PDEs was first introduced by Kansa in 1990, he has used the multiquadric RBF to find the approximate solution of the different types of system of PDEs (Ferreira et al., 2009)

Radial basis function-Pseudospectral method (RBF-PS method) is a well-known numerical technique for solving PDEs. This method was originally developed and used by mathematicians for solving problems in physics. RBF-PS method is a semi-discrete method which is convenient and quite reliable. In this method by discretizing the spatial derivatives only and leaving time variable continuous, the original PDE is converted into a system of ordinary differential equations (ODEs), which is then integrated in time (Eilbeck and Manoranjan, 1986; Fasshauer, 2007). RBFs contain a free shape parameter we will see, which affects the accuracy of a solution and conditioning of RBF interpolation matrix (Platte and Driscoll, 2006; Sarra, 2006).

Here, we consider the Hirota-Satsuma coupled KdV system, which is as follows (Fan, 2001; Manaa and Azzo, 2022):

$$u_t = \frac{1}{2}u_{xxx} - 3uu_x + 3vw_x + 3wv_x$$
$$v_t = -v_{xxx} + 3uv_x \quad (1)$$
$$w_t = -w_{xxx} + 3uw_x,$$

where $u = u(x,t), v = v(x,t)$ and $w = w(x,t)$.

Our aim in this paper is to extend the application of RBF-PS method for finding the approximate solutions of nonlinear Hirota Satsuma coupled KdV system, and we will solve the system on the domain $a \leq x \leq b$, and $0 \leq t \leq T$. We take $x_i = a + i\Delta x$, for $i = 0,\dots,n$, and $t_q = q\Delta t$, $q = 0,\dots,m$, with $\Delta x = (b-a)/n$ and $\Delta t = T/m$.

2- RBF-PSMETHOD

The basic idea of the RBF-PS method is to use a set of smooth basis functions $B_j, j = 0,\dots,n$, such as polynomials to represent the approximate solution of the PDEs (Fasshauer, 2007; Fornberg, 1998; Trefethen, 2000). Thus, the radial base function approximations for the system (1) are given by the following formulas:

$$u(x,t) \approx \hat{u}(x,t) = \sum_{j=0}^{n} c_{1j}\, B_j(x)$$
$$v(x,t) \approx \hat{v}(x,t) = \sum_{j=0}^{n} c_{2j}\, B_j(x) \qquad (2)$$
$$w(x,t) \approx \widehat{w}(x,t) = \sum_{j=0}^{n} c_{3j}\, B_j(x).$$

Here,
$$B_j(x) = \varphi(\|x - x_j\|),$$

where φ is one of the strictly positive definite RBF, and we adopt the Euclidean norm $\|.\|$ to denote the distance between the point x_j and x. The time variable in the formulas (2) ignored (i.e. c_j are unknown time-dependent functions). Thus, only the initial value variable, typically the time in a physical problem, remains. Some most commonly used RBFs are as follows (Buhmann, 2004; Fasshauer, 2007): Multiquadric (MQ): $\varphi(r) = \sqrt{1 + (\varepsilon r)^2}$; and Gaussian (GA): $\varphi(r) = e^{-(\varepsilon r)^2}$, where $r = \|x\|$ is a radial variable and the positive parameter ε is well-known shape parameter used to scale the basis functions.

Now, evaluate (2) at the grid points x_i, for each $i = 0,\dots,n$, we get:

$$\hat{u}(x,t) = \sum_{j=0}^{n} c_{1j}\, \varphi(\|x_i - x_j\|)$$
$$\hat{v}(x,t) = \sum_{j=0}^{n} c_{2j}\, \varphi(\|x_i - x_j\|)$$
$$\widehat{w}(x,t) = \sum_{j=0}^{n} c_{3j}\, \varphi(\|x_i - x_j\|),$$

or, in the matrix-vector notation, we have:

$$U = AC_1$$
$$V = AC_2 \qquad (3)$$
$$W = AC_3,$$

where

$$C_l = [c_{l0}, \dots, c_{ln}]^T, \; l = 1,2,3.$$

and,

$$U = [\hat{u}(x_0,t), \dots, \hat{u}(x_0,t)]^T$$
$$V = [\hat{v}(x_0,t), \dots, \hat{v}(x_0,t)]^T$$
$$W = [\widehat{w}(x_0,t), \dots, \widehat{w}(x_0,t)]^T,$$

and the evaluation matrix A has the form:

$$(A)_{ij} = \varphi(\|x - x_j\|)\big|_{x=x_i}.$$

Now, computing the derivative of $\hat{u}, \hat{v}$, and $\widehat{w}$ in (2) by differentiating the basis functions, as follows:

$$\frac{\partial}{\partial x}\hat{u}(x,t) = \sum_{j=0}^{n} c_{1j}\ \frac{d}{dx}\varphi(\|x - x_j\|)$$
$$\frac{\partial}{\partial x}\hat{v}(x,t) = \sum_{j=0}^{n} c_{2j}\ \frac{d}{dx}\varphi(\|x - x_j\|) \qquad (4)$$
$$\frac{\partial}{\partial x}\widehat{w}(x,t) = \sum_{j=0}^{n} c_{3j}\ \frac{d}{dx}\varphi(\|x - x_j\|).$$

Evaluate (4) at the grid points x_i , as follows:

$$\left.\frac{\partial}{\partial x}\hat{u}(x,t)\right|_{x=x_i} = \sum_{j=0}^{n} c_{1j}\ \frac{d}{dx}\varphi(\|x - x_j\|)\big|_{x=x_i}$$
$$\left.\frac{\partial}{\partial x}\hat{v}(x,t)\right|_{x=x_i} = \sum_{j=0}^{n} c_{2j}\ \frac{d}{dx}\varphi(\|x - x_j\|)\big|_{x=x_i}$$
$$\left.\frac{\partial}{\partial x}\widehat{w}(x,t)\right|_{x=x_i} = \sum_{j=0}^{n} c_{3j}\ \frac{d}{dx}\varphi(\|x - x_j\|)\big|_{x=x_i}.$$

In the matrix-vector notation, we have:

$$U_x = A_x C_1$$
$$V_x = A_x\ C_2 \qquad (5)$$
$$W_x = A_x C_3,$$

where, U, V, W, C_1, C_2 and C_3 are as before, and the derivative matrix A_x has the form:

$$(A_x)_{ij} = \frac{d}{dx}\varphi(\|x - x_j\|)\big|_{x=x_i}.$$

According to the discussion in (Fasshauer, 2007, Trefethen, 2000), if we use strictly positive definite RBFs, then the evaluation matrix A is invertible for any set of distinct collocation points. Also, the non-singularity of the evaluation matrix A depends on the properties of RBFs used, according to (Fasshauer, 2007) the matrix A is positive definite for some RBFs, this fact ensures the non-singularity of the matrix A for distinct supporting points. For more details about the invertibility of the evaluation matrix A see the following references (Fornberg, 1998; Micchelli, 1986; Platte and Driscoll, 2005).

Now, we can solve (3) for the coefficient vectors C_1, C_2 and C_3, as follows:

$$C_1 = A^{-1}U$$
$$C_2 = A^{-1}V \qquad (6)$$
$$C_3 = A^{-1}W.$$

Substitute (6) into (5), we get:

$$U_x = A_x A^{-1}U$$
$$V_x = A_x A^{-1}V$$
$$W_x = A_x A^{-1}W.$$

Define the differentiation matrix D_x, as follows:

$$D_x = A_x A^{-1}.$$

We can write the above equations in the following forms:

$$U_x = D_x U$$
$$V_x = D_x V \quad (7)$$
$$W_x = D_x W.$$

Again, it is possible to find the differentiation matrix concerning the third derivatives, as follows:

$$U_{xxx} = D_{xxx} U$$
$$V_{xxx} = D_{xxx} V \quad (8)$$
$$W_{xxx} = D_{xxx} W,$$

where

$$D_{xxx} = A_{xxx} A^{-1},$$

and the matrix A_{xxx} has the form:

$$(A_{xxx})_{ij} = \frac{d^3}{dx^3} \varphi(\|x - x_j\|)\big|_{x=x_i}.$$

In order to solve the model (1), we will discretize the spatial domain in the model with the collocation points x_i, for each $i = 0, \dots, n$, to obtain:

$$u_t(x_i,t) = \frac{1}{2} u_{xxx}(x_i,t) - 3u(x_i,t)u_x(x_i,t) + 3v(x_i,t)w_x(x_i,t) + 3w(x_i,t)v_x(x_i,t)$$

$$v_t(x_i,t) = -v_{xxx}(x_i,t) + 3u(x_i,t)v_x(x_i,t) \qquad w_t(x_i,t) = -w_{xxx}(x_i,t) + 3u(x_i,t)w_x(x_i,t).$$

In more compact form, we have:

$$\frac{dU}{dt} = \frac{1}{2} U_{xxx} - 3U * U_x + 3V * W_x + 3W * V_x$$
$$\frac{dV}{dt} = -V_{xxx} + 3U * V_x \quad (9)$$
$$\frac{dW}{dt} = -W_{xxx} + 3U * W_x,$$

where $U, V,$ and W are as before, and the symbol $*$ denotes component by component multiplication of two vectors. Substitute equations (7) and (8) into the equation (9), we get:

$$\frac{dU}{dt} = \frac{1}{2} D_{xxx} U - 3U * (D_x U) + 3V * (D_x W) + 3W * (D_x V)$$
$$\frac{dV}{dt} = -D_{xxx} V + 3U * (D_x V) \quad (10)$$
$$\frac{dW}{dt} = -D_{xxx} W + 3U * (D_x W).$$

For simplicity we can write (10) in the following forms:

$$\frac{dU}{dt} = F_1(t, U, V, W)$$
$$\frac{dV}{dt} = F_2(t, U, V, W) \quad (11)$$
$$\frac{dW}{dt} = F_3(t, U, V, W),$$

where,

$$F_1(t,U,V,W) = \frac{1}{2}D_{xxx}U - 3U*(D_xU)$$
$$+3V*(D_xW) + 3W*(D_xV)$$
$$F_2(t,U,V,W) = -D_{xxx}V + 3U*(D_xV)$$
$$F_3(t,U,V,W) = -D_{xxx}W + 3U*(D_xW).$$

The system (1) has become a system of ODEs given by (11) with independent variable t. In this paper, we will solve the system (11) by Runge-Kutta 4th order method (RK4M) (Burden and Faires, 2001).

3. Runge-Kutta 4th Order Method (RK4M)

The most widely used methods for solving ODEs are the series of methods called Runge-Kutta method (Constantinides and Mostoufi, 1999; Golub and Ortega, 1992; Haribaskaran, 2009). The RK4M of (11) is given by:

$$\begin{aligned} U^{(q+1)} &= U^{(q)} + \frac{\Delta t}{6}(J_1 + 2J_2 + 2J_3 + J_4) \\ V^{(q+1)} &= V^{(q)} + \frac{\Delta t}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\ W^{(q+1)} &= W^{(q)} + \frac{\Delta t}{6}(L_1 + 2L_2 + 2L_3 + L_4), \end{aligned} \qquad (12)$$

where,

$$J_1 = F_1\left(t_q, U^{(q)}, V^{(q)}, W^{(q)}\right)$$
$$K_1 = F_2\left(t_q, U^{(q)}, V^{(q)}, W^{(q)}\right)$$
$$L_1 = F_3\left(t_q, U^{(q)}, V^{(q)}, W^{(q)}\right)$$

$$J_2 = F_1\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_1}{2}, V^{(q)} + \frac{\Delta t K_1}{2}, W^{(q)} + \frac{\Delta t L_1}{2}\right)$$

$$K_2 = F_2\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_1}{2}, V^{(q)} + \frac{\Delta t K_1}{2}, W^{(q)} + \frac{\Delta t L_1}{2}\right)$$

$$L_2 = F_3\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_1}{2}, V^{(q)} + \frac{\Delta t K_1}{2}, W^{(q)} + \frac{\Delta t L_1}{2}\right)$$

$$J_3 = F_1\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_2}{2}, V^{(q)} + \frac{\Delta t K_2}{2}, W^{(q)} + \frac{\Delta t L_2}{2}\right)$$

$$K_3 = F_2\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_2}{2}, V^{(q)} + \frac{\Delta t K_2}{2}, W^{(q)} + \frac{\Delta t L_2}{2}\right)$$

$$L_3 = F_3\left(t_q + \frac{\Delta t}{2}, U^{(q)} + \frac{\Delta t J_2}{2}, V^{(q)} + \frac{\Delta t K_2}{2}, W^{(q)} + \frac{\Delta t L_2}{2}\right)$$

$$J_4 = F_1\left(t_q + \Delta t, U^{(q)} + \Delta t J_3, V^{(q)} + \Delta t K_3, W^{(q)} + \Delta t L_3\right)$$

$$K_4 = F_2\left(t_q + \Delta t, U^{(q)} + \Delta t J_3, V^{(q)} + \Delta t K_3, W^{(q)} + \Delta t L_3\right)$$

$$L_4 = F_3\left(t_q + \Delta t, U^{(q)} + \Delta t J_3, V^{(q)} + \Delta t K_3, W^{(q)} + \Delta t L_3\right),$$

and,

$$U^{(q)} = \left[\hat{u}\left(x_0, t_q\right), \dots, \hat{u}\left(x_n, t_q\right)\right]^T$$
$$V^{(q)} = \left[\hat{v}\left(x_0, t_q\right), \dots, \hat{v}\left(x_n, t_q\right)\right]^T$$
$$W^{(q)} = \left[\widehat{w}\left(x_0, t_q\right), \dots, \widehat{w}\left(x_n, t_q\right)\right]^T.$$

Therefore, the numerical solution U, V and W at (x_i, t_q), can be obtained by computing

$U^{(q+1)}$, $V^{(q+1)}$ and $W^{(q+1)}$ in equation (12), successively for each $q = 0, \ldots, m-1$, while $U^{(0)}$, $V^{(0)}$ and $W^{(0)}$ are known by initial conditions.

4. NUMERICAL EXAMPLES

In this section, we will solve the Hirota-Satsuma coupled KdV system (1) over the domain $-10 \le x \le 10$ and in the time period $0 \le t \le 1$, with $n = m = 40$. The domain and time period are discretized with values $\Delta x = 0.025$ and $\Delta t = 0.5$, respectively. The exact solutions are given in (Fan, 2001; Manaa and Azzo, 2022) as:

$$u(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2[k(x + \beta t)]$$
$$v(x,t) = \frac{-4k^2 c_0(\beta + k^2)}{3c_1^2} + \frac{4k^2(\beta + k^2)}{3c_1} \tanh[k(x + \beta t)]$$
$$w(x,t) = c_0 + c_1 \tanh[k(x + \beta t)].$$

The initial conditions are taken from the exact solutions at the value $t = 0$. In this example, we will take $c_0 = c_1 = 1$, and the different values of β and k, as we shall in the tables, also we chose Multiquadric radial base function with $\varepsilon = 5$. The accuracy of the methods is tested by computing the absolute error L_{abs}, which is defined as in (Suarez and Morales, 2014; Yao et al., 2012) by the following formula:

$$L_{abs}(u) = |u(x_i,t) - \hat{u}(x_i,t)|, \text{ for } i = 0,1,\ldots,n.$$

Similarly for v and w. The results are listed in Tables (1-2), and plotted in in Figures (1-12).

Table (1): L_{abs} errors of u, v and w with $k = \beta = 0.3$.

RBF-PS method with RK4M when $t = 0.1$			
x	$L_{abs}(u)$	$L_{abs}(v)$	$L_{abs}(w)$
-10	1.625803E-03	7.651306E-05	1.764379E-03
-8	4.636744E-05	4.357988E-05	5.763666E-04
-6	7.300943E-05	2.742393E-05	5.414266E-04
-4	2.798492E-05	5.064319E-05	1.081821E-03
-2	4.914614E-04	1.657978E-05	3.541536E-04
0	1.726500E-05	1.510738E-04	3.228072E-03
2	5.196478E-04	8.761770E-06	1.873414E-04
4	2.638289E-05	4.924494E-05	1.051783E-03
6	8.047541E-05	2.527336E-05	5.364964E-04
8	8.706604E-05	5.414393E-06	2.547320E-04
10	1.337813E-03	1.697510E-04	3.470836E-03
RBF-PS method with RK4M when $t = 0.5$			
x	$L_{abs}(u)$	$L_{abs}(v)$	$L_{abs}(w)$
-10	1.291636E-02	7.416971E-04	1.719943E-02
-8	7.597353E-04	2.892596E-04	4.336210E-03
-6	4.874436E-04	4.378023E-04	3.498656E-03
-4	2.256570E-04	3.775140E-04	3.252460E-03
-2	2.149724E-03	1.549097E-04	4.132994E-03
0	4.230617E-04	7.332216E-04	1.574830E-02
2	2.856556E-03	2.330868E-05	4.905379E-04
4	1.352488E-04	2.345842E-04	4.925054E-03
6	7.445176E-04	1.663379E-04	1.398544E-03
8	1.339635E-03	1.334669E-04	9.697029E-04
10	6.111996E-03	1.940451E-03	3.960035E-02

Table (2):- L_{abs} errors of u, v and w with $k = \beta = 0.01$.

RBF-PS method with RK4M when $t = 0.1$			
x	$L_{abs}(u)$	$L_{abs}(v)$	$L_{abs}(w)$
-10	*1.146707E-06*	*4.120175E-09*	*3.738279E-03*
-8	*1.213511E-07*	*7.017837E-10*	*6.373474E-04*
-6	*1.701357E-09*	*1.833110E-11*	*1.689398E-05*
-4	*3.671653E-10*	*1.345143E-13*	*1.299816E-07*
-2	*1.008383E-10*	*2.242043E-13*	*4.646379E-08*
0	*4.108094E-13*	*1.479061E-13*	*1.088444E-07*
2	*1.025620E-10*	*6.211341E-14*	*1.667066E-07*
4	*3.491234E-10*	*1.733129E-13*	*3.114441E-07*
6	*2.607272E-09*	*2.454693E-12*	*1.697263E-06*
8	*4.131832E-07*	*9.993427E-12*	*6.294991E-06*
10	*4.164628E-06*	*2.604897E-10*	*1.563602E-04*
RBF-PS method with RK4M when $t = 0.5$			
x	$L_{abs}(u)$	$L_{abs}(v)$	$L_{abs}(w)$
-10	*2.092035E-05*	*2.922510E-08*	*2.646321E-02*
-8	*7.581248E-06*	*6.120103E-09*	*5.540144E-03*
-6	*1.291634E-07*	*6.000083E-09*	*5.443532E-03*
-4	*1.634263E-10*	*1.782701E-09*	*1.618999E-03*
-2	*3.108116E-10*	*2.684503E-10*	*2.449023E-04*
0	*1.115498E-10*	*2.481693E-11*	*2.374474E-05*
2	*4.405236E-09*	*8.509362E-13*	*2.052510E-06*
4	*1.215869E-07*	*3.727857E-12*	*1.215886E-06*
6	*1.869543E-06*	*3.157298E-11*	*1.858072E-05*
8	*1.233029E-05*	*1.020920E-08*	*6.212738E-03*
10	*3.943915E-05*	*2.723420E-08*	*1.656146E-02*

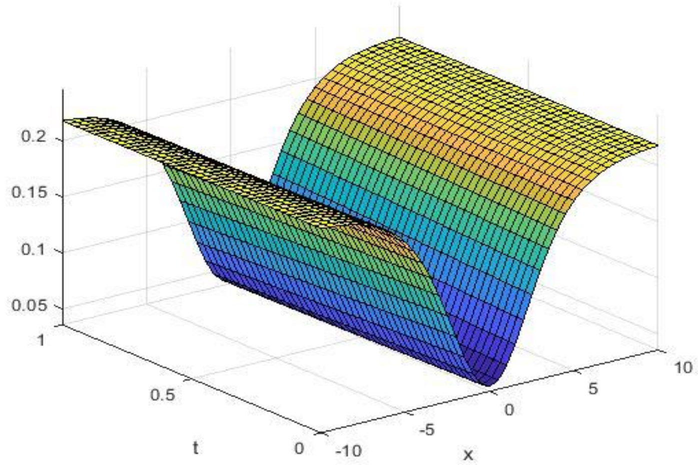

Fig. (1): -Exact solution u for KdV system when $-10 \leq x \leq 10$ and $0 \leq t \leq 1$ with $c_0 = c_1 = 1,\ k = \beta = 0.3$, and $n = m = 40$.

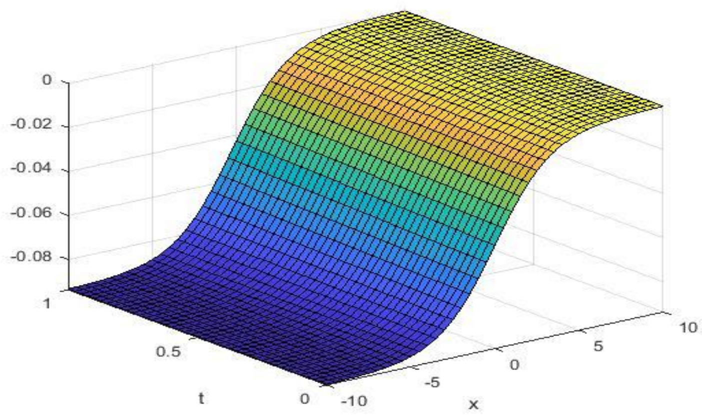

Fig. (2):- Exact solution v for KdV system. The rest of the parameters and the domains are the same as in Figure 1.

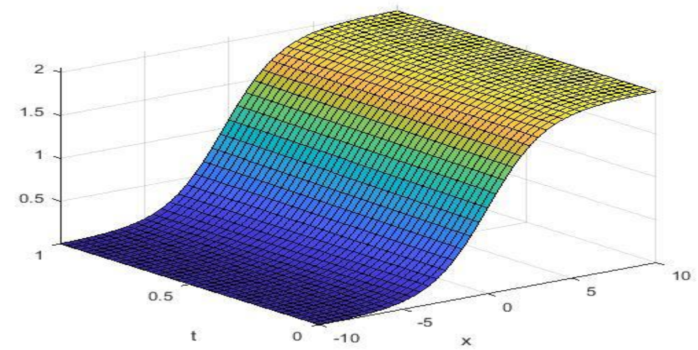

Fig. (3):- Exact solution w for KdV system. The rest of the parameters and the domains are the same as in Figure 1.

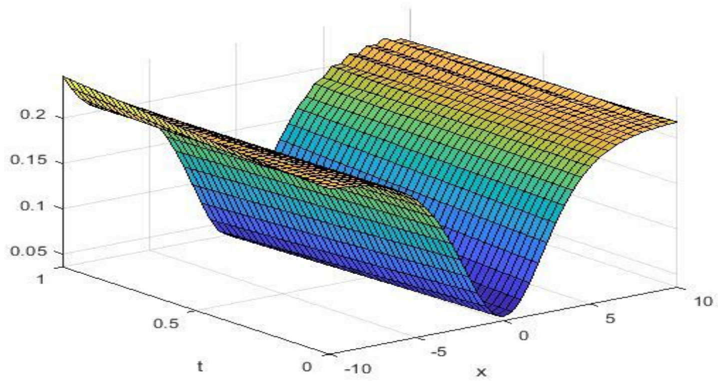

Fig. (4):- Approximate solution $\hat{u}$ for KdV system obtained by RBF-PS method with Multiquadric radial base function when $\varepsilon = 5$. The rest of the parameters and the domains are the same as in Figure 1.

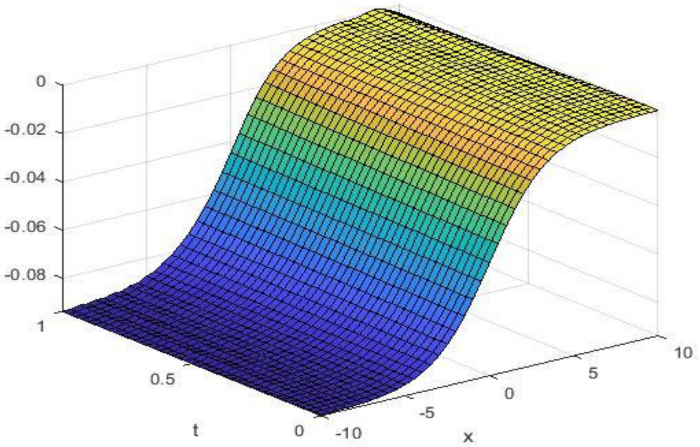

Fig. (5): -Approximate solution $\hat{v}$ for KdV system obtained by RBF-PS method with Multiquadric radial base function when $\varepsilon = 5$. The rest of the parameters and the domains are the same as in Figure 1

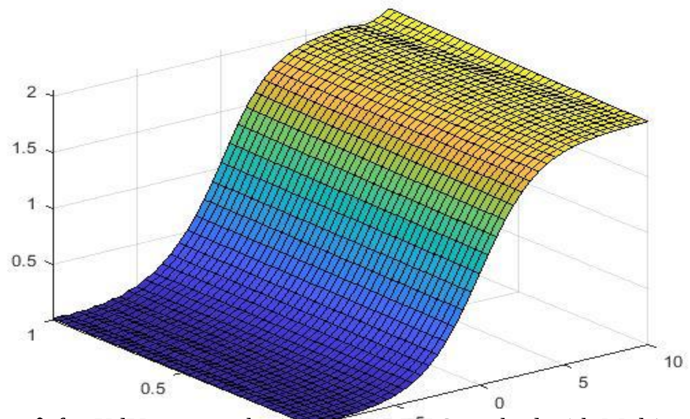

Fig. (6):- Approximate solution $\hat{w}$ for KdV system obtained by RBF-PS method with Multiquadric radial base function when $\varepsilon = 5$. The rest of the parameters and the domains are the same as in Figure 1.

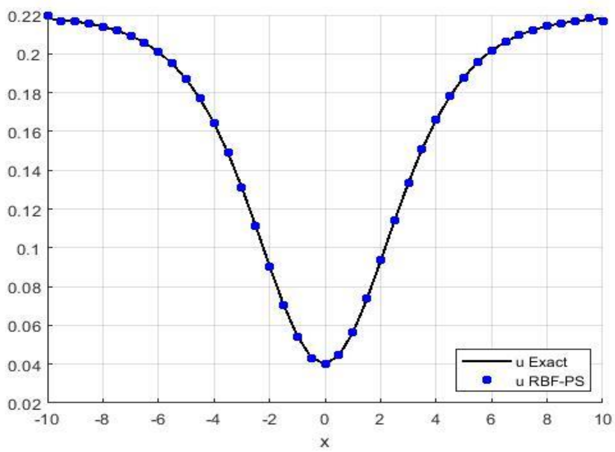

Fig. (7):- Comparison between approximate solution $\hat{u}$ and the exact solution u when $t = 0.1$. The rest of the parameters and the x −domain are the same as in Figure 1.

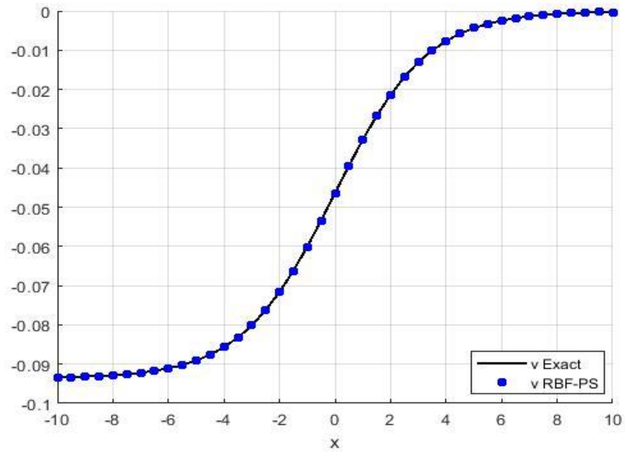

Fig. (8):- Comparison between approximate solution $\hat{v}$ and the exact solution v when $t = 0.1$. The rest of the parameters and the x −domain are the same as in Figure 1.

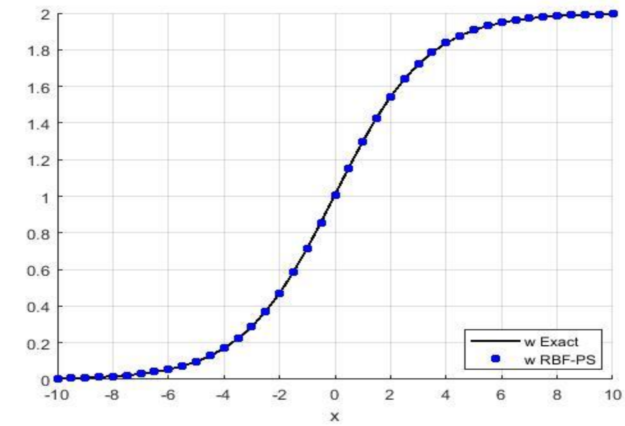

Fig. (9):- Comparison between approximate solution $\hat{w}$ and the exact solution w when $t = 0.1$. The rest of the parameters and the x −domain are the same as in Figure 1.

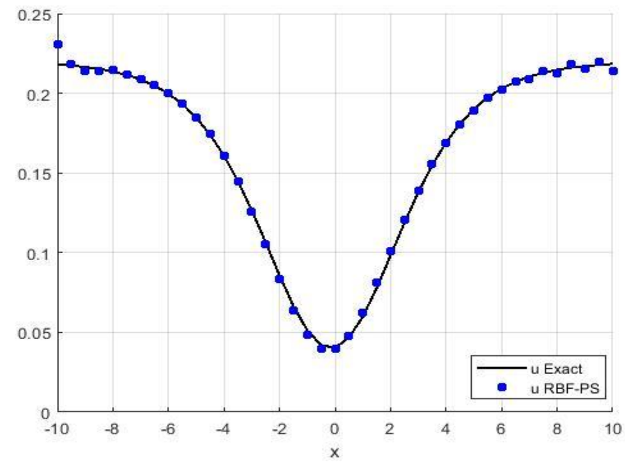

Fig. (10):- Comparison between approximate solution $\hat{u}$ and the exact solution u when $t = 0.5$. The rest of the parameters and the x −domain are the same as in Figure 1.

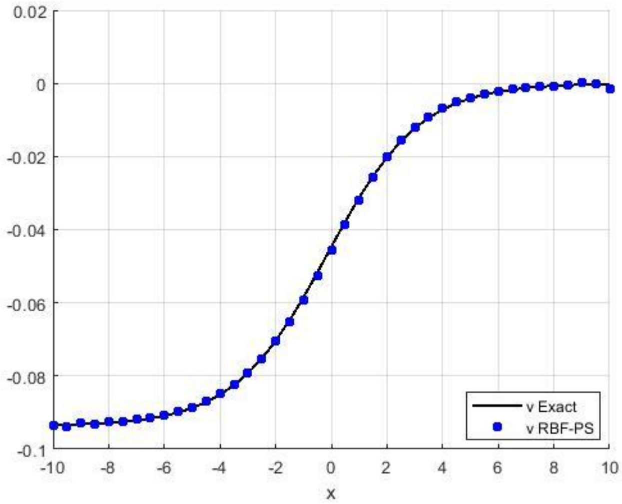

Fig. (11):- Comparison between approximate solution $\hat{v}$ and the exact solution v when $t = 0.5$. The rest of the parameters and the x −domain are the same as in Figure 1.

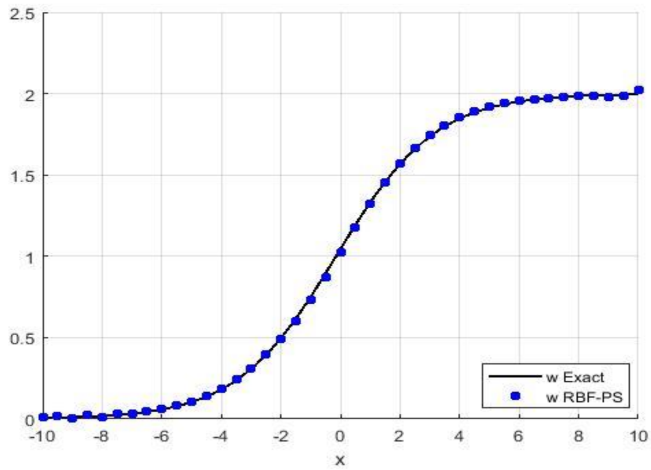

Fig. (12):- Comparison between approximate solution $\hat{w}$ and the exact solution w when $t = 0.5$. The rest of the parameters and the x −domain are the same as in Figure 1

5. CONCLUSION

We have solved the Hirota-Satsuma coupled KdV system (1) numerically, by using RBF-PS method. The numerical results show that this method is a powerful and efficient technique for finding the approximate solutions of the model. The results of the present method are in excellent agreement with the exact solutions. In general, this method solves the KdV system accurately. In particular, this method has optimal results for some values of parameters.

6. ACKNOWLEDGMENTS

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7. REFERENCES

Buhmann, M. D., (2004). Radial Basis Functions: Theory and Implementations. Cambridge University Press.

Burden, R. L. and Faires, J. D. (2011). Numerical Analysis. Ninth Edition. Brooks/Cole, Cengage Learning.

Constantinides, A. and Mostoufi, N. (1999). Numerical Methods for Chemical Engineers with MATLAB Applications. Upper Saddle River, NJ: Prentice Hall PTR.

Eilbeck, J. C. and Manoranjan, V. S. (1986). A comparison of basis functions for the pseudo-spectral method for a model reaction-diffusion problem. Journal of Computational and Applied Mathematics, 15(3), 371-378.

Fan E. (2001). Soliton Solutions for a Generalized Hirota-Satsuma Coupled KdV Equation and a Coupled MKdV Equation. Physics Letters A, Vol. 2852, No. 1-2, 18-22.

Fasshauer, G. E. (2007). Meshfree Approximation Methods with MATLAB. Interdisciplinary Mathematical Sciences, Vol.6. World Scientific Publishing, Singapore.

Ferreira, A. J.; Kansa, E. J.; Fasshauer, G. E. and Leitao, V.M.A. (2009). Progress on Meshless Methods.
Springer Science and Business Media.

Fornberg, B. (1998). A Practical Guide to Pseudo-pectral Methods. Cambridge University Press.

Gao, Y.T.; Tian, B. (2021) Ion-acoustic shocks in space and laboratory dusty plasmas: Two-dimensional
and non-traveling-wave observable effects. Phys. Plasmas , 8, 3146.

Golub, G. H. and Ortega, J. M. (1992). Scientific Computing and Differential Equations: An Introduction to Numerical Methods. Academic Press.

Haribaskaran, G. (2009). Numerical Methods. 2nd edition. University Science Press.

Jain, M. K. (1979). Numerical Solution of Differential Equations. John Wiley and Sons, New York.

Mahmoud A. E. Abdelrahman; M. B. Almatrafi and Abdulghani Alharbi (2020). Fundamental Solutions for the Coupled KdV System and Its Stability. Symmetry. Vol. 12.

Manaa S.A., and Azzo Sh. M. (2022). Sumudu-Decomposition Method to Solve Generalized Hirota-Satsuma Coupled Kdv System. Science Journal of University of Zakho. 10(2), 43-47.

Micchelli, C. A. (1986). Interpolation of scattered data: distances, matrices, and conditionally positive
definite functions. Const. Approx. 2, 11-22.

Platte, R. B. and Driscoll, T. A. (2005). Polynomials and potential theory for Gaussian radial basis function interpolation. SIAM Journal on Numerical Analysis, 43(2), 750-766.

Platte, R. B. and Driscoll, T. A. (2006). Eigenvalue stability of radial basis function discretizations for
time dependent problems. Computers and Mathematics with Applications, 51(8), 1251-1268.

Sarra, S. A. (2006). Integrated multiquadric radial basis function approximation methods. Computers and
Mathematics with Applications, 51(8), 1283-1296.

Suarez, P. U. and Morales, J. H. (2014). Numerical solutions of two-way propagation of nonlinear dispersive waves using radial basis functions. International Journal of Partial Differential Equations, 2014, 1-8.

Trefethen, L. N. (2000). Spectral Methods in MATLAB. SIAM, Philadelphia.

Yao, G.; Siraj-ul-Islam and Sarler, B. (2012). Assessment of global and local meshless methods based on collocation with radial basis functions for parabolic partial differential equations in three dimensions. Engineering Analysis with Boundary Elements, 36(11), 1640-1648.

Yucel C.; Ali K. and Orkun T. (2017). On the New Solutions of the Conformable Time Fractional Generalized Hirota-Satsuma Coupled KdV System. Seria Matematica – Informatica. Vol. 1. 37-49.