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IDENTIFYING THE BEST-FIT PROBABILITY DISTRIBUTION MODEL TO PREDICT THE ANNUAL MAXIMUM DAILY RAINFALL IN DUHOK CITY, IRAQ

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ABSTRACT

Adequate knowledge of extreme events design for a long return period is required for the design and construction of such projects. This study aims to investigate the best-fit probability distribution model to predict extreme rainfall for the 30 years of observed annual maximum daily rainfall data of Duhok city. For the aim of this study, six candidate probability distribution methods were selected namely Normal, Log-Normal, Log-Normal 3p, Log-Pearson type 3, Generalized Extreme Value (GEV), and Gumbel Max. Then these models were subjected to the three goodness-of-fit tests: Chi-Square, Anderson-Darling and Kolmogorov tests. Depending on the lowest summation of ranked scores of each probability distribution model, in the area being researched, the best-fit distribution is chosen. According to the analysis of data, Generalized Extreme Value distribution can be used as best-fit prediction of the maximum daily rainfall in a year followed by Log-Normal 3P and Log-Pearson 3P for Duhok city. Furthermore, the maximum daily rainfall calculated values for the return periods of 2, 5, 10, 25, 50 and 100 years by the GEV distribution formula of return periods were found to be 70 mm, 93mm, 109mm, 130mm, 147 mm and 164mm respectively. The study's findings can be applied to the creation of more precise flood risk and damage models.

KEYWORDS: Rainfall, Probability Distribution, Annual Maximum Daily Rainfall, Goodness-Of-Fit Test

INTRODUCTION

Lakes and rivers serve as critical water sources for humans, animals, and agriculture, relying on rainfall contributions from nature. However, accurately forecasting the future occurrence and quantity of rainfall remains challenging, as highlighted in a study by Amin et al. in 2016. To enhance water resource management and utilization, statistical methods, such as probability distributions, play a crucial role in analyzing rainfall data, particularly considering the highest daily precipitation observed throughout the year, as noted by Subudhi in 2007. Additionally, probability and frequency analysis, as researched by Bhakar et al. in 2008, enable the calculation of projected rainfall for different return periods, offering valuable insights for planning and decision-making processes.

Reliable information about extreme events with long return periods is needed for the management and implementation of water resource strategies to limit flood damages before designing and building hydrologic infrastructures like systems for urban drainage, dykes, reservoirs, dams, and bridges (Tao et al, 2002). In modern times, there exists a wide range of probability distribution models employed to forecast expected rainfall for various return periods. However, a significant challenge faced in engineering practice is the proper selection and identification of the most suitable distribution model (Amin et al., 2016; Tao et al., 2002). Determining the ideal distribution model for a specific site relies on the thorough evaluation of available models to ensure accurate estimations of extreme rainfall events. This crucial task ensures that the chosen distribution model is capable of providing reliable predictions for planning and designing resilient infrastructure and mitigating potential risks associated with extreme weather conditions.

Previously, a number of scientific papers evaluated distribution models in Iraq such as Dawood (2009), Aboodi (2014), Al- Baldawi and Al- Zuabidi (2016) and Omer et al. (2019). In 2009, Dawood conducted a study to assess the appropriate theoretical statistical distribution of extreme monthly rainfall in Mosul, Iraq. For his analysis, he considered five different distribution models: Normal, Pearson Type III, Lognormal, 3-parameter lognormal, and Gumbel. To determine the validity of each distribution model, Dawood employed the Chi-square test. By comparing the observed extreme monthly rainfall data with the expected values generated from each distribution model, he drew his conclusions. Based on the results of the Chi-square test, Dawood's study concluded that all five selected distribution methods (Normal, Pearson Type III, Lognormal, 3-parameter lognormal, and Gumbel) were suitable for estimating maximum monthly rainfall in the city of Mosul. This implies that any of these five probability distribution models can be applied to accurately estimate extreme monthly rainfall in Mosul, according to the findings of Dawood's research. However, it's important to keep in mind that these results specifically pertain to the context of extreme monthly rainfall in Mosul and may not be directly applicable to other regions or different types of rainfall patterns. Local climatic factors and data should be considered when selecting an appropriate distribution model for rainfall analysis and prediction in other areas. Furthermore, in 2019, Omer and his colleagues conducted a study with the aim of identifying the best-fit probability distribution to determine the most frequent rainfall amount in each month for three cities in Iraq: Erbil, Sulaimani, and Duhok. During their investigation, the researchers analyzed the rainfall data for each city and tested various probability distribution models to find the most suitable one for each month. However, what they found was that there is no common best-fitted model that applies uniformly to all months in any of the three cities. This result indicates that the rainfall patterns in Erbil, Sulaimani, and Duhok exhibit considerable variability and complexity throughout the year. Different months within each city may experience distinct weather patterns, leading to varying rainfall behaviors. Consequently, a single probability distribution model cannot adequately capture the diverse rainfall characteristics observed in all the months for any of the three cities.

Despite several papers evaluating Duhok city's rainfall data, none of them have conclusively determined the most suitable distribution models to use for this specific location. To address this gap, the current study aims to utilize the available 30 years of data to assess various probability distribution models. The objective is to identify the best-fitting models for predicting extreme rainfall events in Duhok city, particularly during different return periods. By conducting this analysis, the study seeks to provide valuable insights into the probability distribution patterns of rainfall in the region, aiding in more accurate and reliable predictions of extreme weather events in the future.

As previously mentioned, selecting the most appropriate probability distribution for rainfall data lacks a universal guideline. To address this issue, the current study proposes three techniques: the Kolmogorov-Smirnov Test, Chi-Squared Test, and Anderson-Darling Test. These methods will be employed to evaluate their suitability in fitting the data. Specifically, six candidate theoretical probability distributions are tested: Normal, Log-Normal, Lognormal 3P, Log-Pearson type 3, Generalized Extreme Value, and Gumbel Max. These distributions will be assessed for their ability to fit the maximum annual daily rainfall data recorded in Duhok city. By employing these techniques and evaluating multiple distribution options, the study aims to identify the best-fitting model that can accurately predict extreme rainfall events during various return periods in Duhok city.

DATA AND THE STUDY AREA

The city of Duhok is located in northwestern Iraq, in the western part of the Kurdistan region. It is located at 36° 52' north latitude and 43°59' east longitude. It is at an altitude of 540m above sea level (Figure 1). Two mountain ranges surround the city of Duhok: Bekher to the north and northeast and Zawa to the southeast. A city with two rivers runs through it (i.e. . The Duhok River and Heshkarow River). Both rivers converge in the southwestern part of the city, and the water is used primarily for irrigation purposes and helps maintain nearby green spaces.

Fig. (1): Map of Iraq with Dohuk city indicated in black circle in the far north

Duhok city has a Mediterranean climate (*Csa*), According to the Köppen-Geiger climate classification system, it has hot summers with almost no rain and cool to cold and wet winters (Britannica Encyclopedia, 2019). Rainfall decreases during the colder months, heaviest in late winter and early spring. The city can expect about two or three days of snowfall per year, with more snowfall in the highlands (Mohammed, 2010).

For this study, the daily rainfall data for the Duhok city were used. The provided data was observed and recorded by Duhok Dam Directorate. These data were containing the total daily rainfall in millimeters received during the period of the 1991-2020 year. For analysis the annual maximum daily rainfall (AMDR) peaked out for data spanned between 1991 and 2020 (Table 1).

Table (1): The annual maximum daily rainfall in Duhok City for the years 1991 to 2020

Year	Max. daily rainfall (mm)	Year	Max. daily rainfall (mm)
1991	25	2006	65
1992	102	2007	36
1993	52	2008	35
1994	60	2009	70
1995	102	2010	36
1996	40	2011	69
1997	47	2012	51
1998	33	2013	89
1999	35	2014	121
2000	49	2015	29
2001	38	2016	52
2002	86	2017	40
2003	55	2018	59
2004	66	2019	74
2005	55	2020	85

METHODOLOGY

Certain parameters can be used to describe the statistical behavior of any hydrological series. Gupta and Kapoor (2002) used commonly used statistical analysis procedures, which were followed herein. As measures of variability of hydrological series, the calculations of statistical parameters should be done. These parameters include mean, standard deviation, coefficient of variation, and coefficient of skewness. In the current study, all of the parameters were used to describe the variability of rainfall.

1. Empirical frequency analysis

The return periods of the AMDR events for Duhok city were determined using an empirical continuous probability distribution function (ECPDF) for the period 1991-2020.

The average amount of time it takes for each extreme event of a particular size to be equaled or exceeded at least once is known as the return period, also known as the recurrence interval (Patra, 2001). By placing the maximum daily rainfall for one day in decreasing order and determining their respective ranks, the return period (T) was calculated using Weibull's plotting position formula (Chow, 1964):

$$T = \frac{N+1}{M} \ldots\ldots\ldots\ldots\ldots \quad 1$$

where

N is the overall number of record years;

M denotes the observed rainfall values in decreasing order.

2. Selection of the probability distribution models

To calculate the quantiles of AMDR occurrences with up to 30-year recurrence periods, the theoretical continuous probability distribution function (TCPF) was used. By initially locating TCPDFs that closely matched AMDR events, quantile estimation was carried out. To select candidates and the best TCPDF for frequency analysis of hydrometeorological occurrences, an adopted methodology should be preceded. In 2018 Masereka et al. devised a methodology and their methodology was employed in this study to identify candidates and optimal TCPDFs for frequency analysis of AMDR events.

Various probability distribution models like Normal, Log-Normal, Lognormal 3P, Log-Pearson type 3, Generalized Extreme Value, and Gumbel Max. were candidate to fit with the AMDR.

Probability distribution analysis was performed according to standard methods (Olofintoye et al., 2009; Nemichandrappa et al., 2010; Sharma and Singh, 2010; Olumide et al., 2013). Each model has its parameters. For example, Normal distribution has μ stand for mean and σ stand for standard deviation. As well as lognormal distribution has two parameters: scale and shape parameter. While lognormal 3p distribution is three-parameter family distribution: shape (α), scale (β) and location (σ) see Table 3. However, k and μ were demonstrated for shape and location parameters in generalized extreme value. The parameter α is demonstrated for shape parameter which is a measure of the distribution's skewness. The scale parameter (β) is a measure of the steepness of the distribution. The smaller the β is, the skewed curve becomes more positive.

3. Pick the theoretical continuous probability distribution functions candidates

Probability-probability (P-P) plots were created from the AMDR data; the P-P plot is a graph plotting the empirical continuous probability distribution functions (ECPDFs) values versus the theoretical continuous probability distribution functions (TCPDFs) values. It is employed to assess how well a particular distribution fits the collected data. If the theoretical distribution given is the right model, then this plot will be roughly linear. The reference diagonal line that the graph points should fall along is shown by EasyFit.

Furthermore, the magnitude and frequency of AMDR events were analyzed by P-P plots to identify the candidate TCPDFs visually. The software EasyFit supports 50 continuous probability distribution functions. It generates 50 P-P graphs by plotting ECPDf values versus TCPDF values for all 50 probability types supported by the named software (Mathewave, 2015). Then, as candidate TCPDFs, the six TCPDF curves that can be seen to be the closest to the plot's diagonal were chosen.

Using quantile-quantile (Q-Q) plots, the six found candidate TCPDFs were verified. A Q-Q plot is constructed by plotting the observed values of AMDR events against those fitted (theoretical) distribution quanta. The units of both axes of Q-Q charts are in units of the observed (input) data set. It can be clarified that, the horizontal axis of this histogram contains the observed data values which are the annual maximum daily rainfall in mm while the values of the vertical axis involved

$$F^{-1}(F_n (X_i)-0.5/n)$$

where

$F^{-1}(X)$ is the inverse cumulative theoretical continuous probability distribution function. $F_n(x)$ is the empirical continuous probability distribution function; and n is a sample size.

If the theoretical distribution given is the right model, the Q-Q plot will be roughly linear. To show where points on the chart will fall, EasyFit displays a diagonal reference line. The spacing of the TCPDF histograms with the diagonal shown in the EasyFit5.5 plot represents the validation of the candidate TCPDFs (Mathwave, 2015).

4. Testing of the Probability Distribution Models

To select best-fit probability distribution among the six candidate models, the performance of the goodness of fit analysis for the distribution models should be applied. Easyfit 5.5 software can examine the data sample by using three commonly employed goodness of fit tests in statics calculations and which are more popular in rainfall analysis. These tests are Kolmogorov-Smirnov (K-S), Anderson- Darling (A^2), and Chi-Squared X^2 test.

To compare, select, and determining the best distribution models, one might utilize goodness-of-fit tests. A mutually supportive report is created by EasyFit software, which aids in gaining a broad perspective on fitted distributions and assessing the level of fit goodness for specific models at various significance levels. These tests are:

(i) Chi-square test

Chi-square (X^2) is a goodness-of-fit test that compares how well the theoretical continuous probability distribution functions (TCPDF) fits the Empirical continuous probability distribution functions (ECPDF). The chi-square statistic is defined as (Olofintoye et al., 2009).

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad \ldots\ldots\ldots\ldots\ldots 2$$

where:

x^2 is the test statistic

O_i is the observed value

E_i is the expected value

where i is a number ranging from 1 to n and n is the number of samples

(ii) Kolmogorov–Smirnov test

In 1998, Kottegoda and Rosso indicated that the Kolmogorov-Smirnov (KS) test is a non-parametric test. It is used to verify whether the used data sample was drawn from a reference or to evaluate if they come from an identical distribution. Finding the biggest absolute variances between the theoretical and cumulative empirical distribution functions yields the test statistic. By mean, if a random sample, $x_1, x_2 \ldots \ldots \ldots x_n$, comes from the same distribution as CDF $F(x_1)$, then KS test statistic:

$$D^+ = \max\left(\frac{i}{n} - F(x_i) \right) \ldots\ldots\ldots\ldots 3$$

and

$$D^- = \max\left(F(x_i) - \frac{i-1}{n} \right) \ldots\ldots\ldots\ldots 4$$

where

D^+ is the maximum positive difference. It identifies the region where the fitted CDF is lower than the empirical CDF and where there is the highest vertical difference between the two CDFs.

D^- is the maximum negative difference. This identifies the region where the fitted CDF is above the empirical CDF and where there is the highest vertical difference between the two CDFs.

(iii) Anderson–Darling test

Anderson–Darling (A^2) is a goodness-of-fit test in which the fitting of an observed continuous PDF (sample) to an expected continuous PDF (parent) is carried out. The test statistic A^2 is defined as (Mzezewa et al., 2010):

$$A^2 = -n - \frac{i}{n}\sum_{i=n}^{n}(2i-1)\left[\ln F(x_i) + \ln\left(1 - F\left(x_{n-i+1}\right)\right)\right] \ldots\ldots 5$$

where

A^2 Anderson darling test.

n the number of events in the sample.

F(x) Cumulative distribution for the specified distribution.

i the i^{th} sample when the data is sorted in ascending order.

5. The best-fit theoretical continuous probability distribution functions (TCPDF)

The total scores from the three goodness tests were used to determine the best distribution model for each dataset. Based on the minimum test statistic, the ranks of various probability distributions were assigned from one to six. The results from the three goodness tests were added up to produce a final score for a particular distribution. The best distribution model for the given time series was determined to be the probabilistic model with the lowest total score. Likewise, the distribution model with the second total lowest score was regarded as the second best one (Chowdhury and Stedinger 1991, Adegboye and Ipingong 1995, Murray and Larry, 2000; Olofintoye et al. 2009).

6. Quantile function

Using the maximum likelihood approach in the EasyFit 5.5 program, estimations of the selected best-fit TCPDF parameters were made. To generate a quantile function (QF), the estimated parameters to the PDF of the discovered optimal TCPDF was used (Mathwave, 2015). The created QF was used to calculate AMDR quantiles for recurrence periods lasting 2, 5, 10, 25, 50 and 100 years.

RESULTS AND DISCUSSION

The annual maximum daily rainfall data for Duhok city were used in this study. These data were analyzed to find the best-fit probability distribution. The parameters of each model were calculated by EasyFit software. The mean value of the observed data sets of AMDR was equal to 58.52mm. The standard deviation and the coefficient of variation were found to be 24.17mm and 0.41 respectively.

Equation 1 is applied to determine the empirical return periods of each annual maximum daily rainfall event for the data sample size of 31 events and is tabulated in Table 2. This table shows the correlation between the AMDR events' plotted positions and the empirical return periods that correspond to them.

Table (2): The empirical return period of each annual maximum daily rainfall

i	X(mm)	P_i	$T=1/P_i$(yr)
1	121.0	0.03	31.00
2	102.0	0.06	15.50
3	102.0	0.10	10.33
4	89.00	0.13	7.75
5	85.98	0.16	6.20
6	85.20	0.19	5.17
7	73.80	0.23	4.43
8	70.00	0.26	3.88
9	69.30	0.29	3.44
10	66.40	0.32	3.10
11	64.80	0.35	2.82
12	60.00	0.39	2.58
13	59.40	0.42	2.38
14	55.28	0.45	2.21
15	54.50	0.48	2.07
16	52.00	0.52	1.94
17	51.50	0.55	1.82
18	50.60	0.58	1.72
19	49.00	0.61	1.63
20	47.00	0.65	1.55
21	39.60	0.68	1.48
22	39.50	0.71	1.41
23	38.40	0.74	1.35
24	36.00	0.77	1.29
25	35.76	0.81	1.24
26	35.40	0.84	1.19
27	35.00	0.87	1.15
28	33.00	0.90	1.11
29	29.20	0.94	1.07
30	25.00	0.97	1.03
31	00.00	1.00	1.00

In this case study, six methods of probability distributions were candidates namely: Normal, Log-Normal, Lognormal 3P, Log-Pearson type 3, Generalised Extreme Value, and Gumbel Max. The parameter of each model was calculated by EasyFit software. The summary of the parameter calculations were shown in Table 3. To find the best-fitted probability distribution model in Duhok city, the goodness-of-fits tests: Kolmogorov Smirnov Test, Chi-Squared Test, and Anderson Darling Test were applied for all the selected probability distributions.

Table (3): Summary of the calculated parameters of probability distributions

NO.	Distribution	Parameters
1	Gen. Extreme Value	$k=0.05278$ $\sigma=18.61$ $\mu=46.757$
2	Gumbel Max	$\sigma=18.846$ $\mu=47.643$
3	Log-Pearson 3	$\alpha=275.31$ $\beta=0.02431$ $\gamma=-2.7027$
4	Lognormal	$\sigma=0.3966$ $\mu=3.9904$
5	Lognormal (3P)	$\sigma=0.52604$ $\mu=3.7007$ $\gamma=12.275$
6	Normal	$\sigma=24.17$ $\mu=58.521$

Earlier, several authors described the application of goodness of fit tests and the standard procedures to be followed (Chowdhury and Stedinger (1991), Adegboye and Ipinyomi (1995), Murray and Larry (2000)).

According to the goodness of fit tests, all the selected probability distributions used in this study were ranked from one (best fit) to six (least fit). The probability distribution model that gets the lowest score (total of all rated goodness of fit test results) is chosen as the model with the best fit. The obtained results of the goodness of fit tests with their rank for each probability distribution model are tabulated in Table 4. The Generalized Extreme Value (GEV) distribution, based on the results of the goodness of fit tests, is the best-fit probability distribution selection for the used metrological data. However, Log-Normal 3p and Log-Pearson 3 distributions got the same sum of rank's score thus they are considered as the second best-fit distributions for the studied area.

Table (4): The total rank of the six probability distributions' goodness-of-fit tests

No.	Distribution	Kolmogorov-Smirnov	Anderson-Darling	Chi-Squared	Total
		Rank	Rank	Rank	
1	Generalized Extreme Value	1	1	2	4
6	Normal	2	6	1	9
3	Log-Pearson 3	3	2	4	9
5	Lognormal (3P)	4	5	6	15
2	Gumbel Max	5	3	3	11
4	Lognormal	6	4	5	15

Despite making the mathematical calculations to realize the differences between the distribution models, graphical tests are of main rule. Graphical tests P-P and Q-Q plots were applied. These plots can be used as a visual method to identify theoretical continuous probability distribution functions for AMDR event frequency analysis. The developed plots are shown in Figures 3 and 4 for the six selected probability distribution models to visually identify the best-fitted model for the particular city chosen for this study. Depending on the P-P plot shown in figure 3 below the candidate TCPDFs whose best close to the diagonal of the plot is GEV.

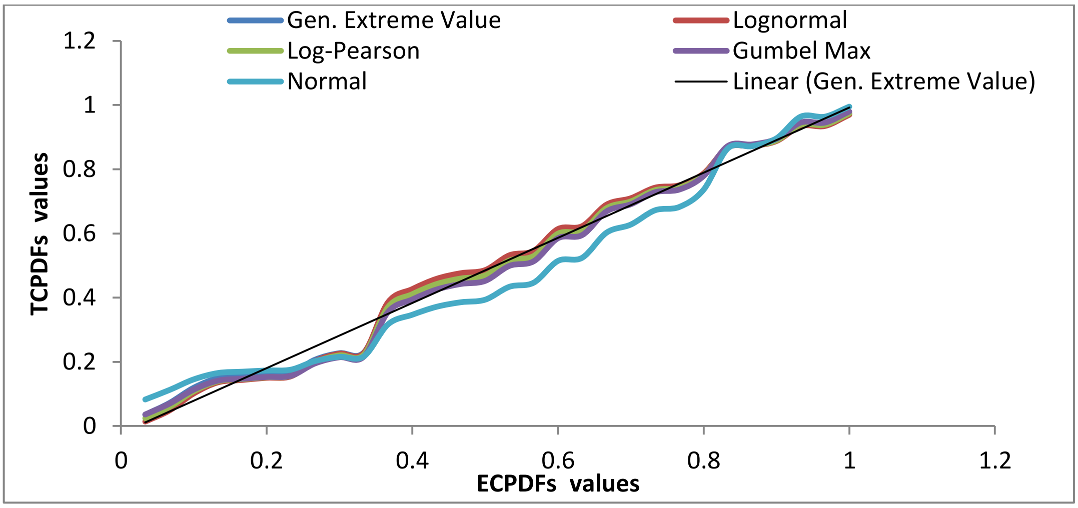

Fig. (3): P-P plots of the six candidate probability distribution models

In addition to the above plots, the Q-Q plots of all six applied distribution models were plotted for the graphical analysis.

Both axes of this graph are in units of the input data set which is millimeter. Figure 4 illustrates the Q-Q plots for the fitted distributions. From the plotted graphs, the best-fitted model can be selected on the base of observing which model data are more likely to be placed on the diagonal line (1:1 line). Based on this visualization the same results of mathematical calculation comparison are noticed which means the theoretical estimation of rainfall by the Generalised Extreme Value (GEV) method is best fitted on the diagonal line and its prediction of theoretical rainfall is very close to those observed and recorded maximum rainfall values.

Generalised Extreme Value probability distribution function

In 1995, Jenkinson proposed the Generalised Extreme Value (GEV) distribution, a distribution of three parameters family (scale, location and shape), as a model for extreme natural phenomena such as rainfall, sea level, and river heights (Dupuis and Field 1998). According to the QF, the GEV can be defined as (Alam et al., 2018):

$$X_T = \mu + \frac{\sigma}{k}\left[\left(-\log\left(1-\frac{1}{T}\right)\right)^{-k} - 1\right] \ldots..6$$

where,

X_T Estimated quantile of the return period (T)

μ location parameter

σ scale parameter

k shape parameter

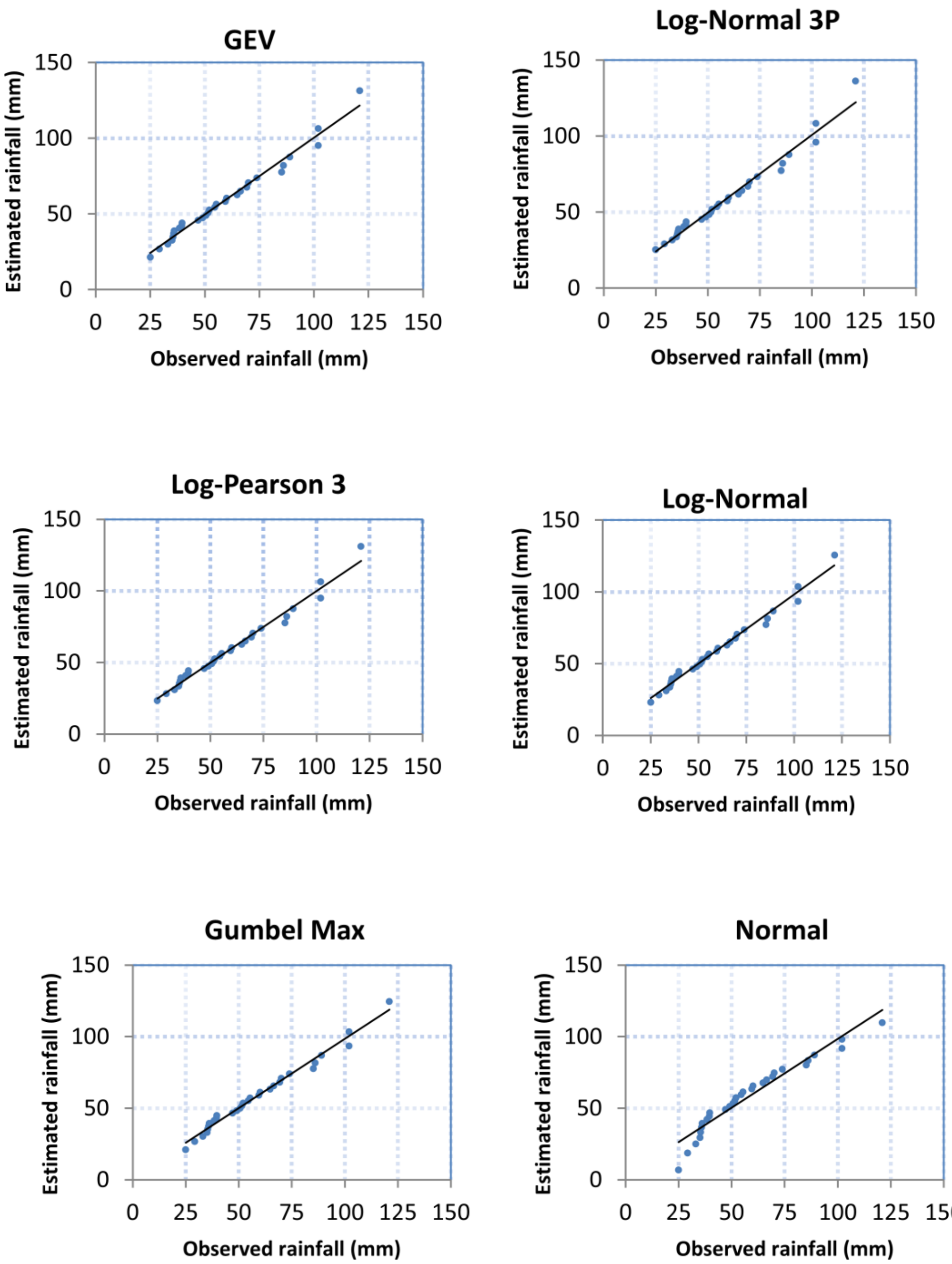

Fig. (4): Quantile-Quantile plots of the candidate theoretical continuous probability distribution models

The parameters of GEV are calculated by Easyfit 5.5 has been shown in Table 2. As identified before a part of the study goals was to calculate the prediction of maximum daily rainfall using the selected best-fit distribution in different return periods. The expected rainfall values of the 2 years, 5 years, 10 years, 25 years, 50 years and 100 years were calculated using equation 6 of the return period of GEV defined by QF and it found to be 70 mm, 93mm, 109mm, 130mm, 147 mm and 164 mm respectively. However, there may be a significant difference in the predicted rainfall values obtained from the GEV for high-return periods. By the mean when considering a return periods of long terms for example 1000 years may lead to over-estimate of the rainfall amount. The results could be better evaluated by including the determination of confidence intervals of the estimated percentiles to show the relative uncertainty at different return periods, but this is not part of our study.

Furthermore, it is important to mention that longer sets of maximum daily rainfall in Duhok city are required to reduce the uncertainty in rainfall quantile estimates for higher return periods that are unavailable in Duhok. The obtained results are useful to be used for future design and the management of projects to control the risk of natural phenomena such as floods.

CONCLUSION

The 30 years observed annual maximum daily rainfall in Duhok city was used in this study. The study aimed to investigate the best-fit probability distribution models that can predict maximum daily rainfall for any year. Based on the scores of goodness-of-fit tests used in this study, the Generalized Extreme Value (GEV) distribution was identified to be the best-fit probability followed by Log-Normal 3P and Log-Pearson 3. The expected values of the maximum daily rainfall for the different return periods were calculated using the selected best-fitted distribution model in this study. These predictions are most important and helpful to design future construction works for the studied area and as well as reducing the risks of natural phenomena such as floods. As a recommendation, longer data sets of observed daily rainfall for the studied area will be helpful to reduce the uncertainty in the Duhok rainfall quantile estimations for longer return periods which are not available at the moment.

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