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# EFFECTS OF LAMINA ORIENTATION ON UNIAXIAL BUCKLING LOAD OF COMPOSITE PLATES

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## ABSTRACT

**The purpose of this research is to look at the buckling behavior of axially loaded, symmetrical laminated composite rectangular plates with simple support boundary conditions all around. In this work, balance equations based on Classical Plate Theory (CPT) produced from Kirchhoff assumptions are used to present formulas that determine the critical buckling load for orthotropic plates step by step. The formulas that provide the critical buckling load for thin composite plates with simple support on all four sides under a uniaxial compression force are presented. For the laminated rectangular composite plate constructed using different fiber orientations of N orthotropic layers employed in analytical solution, the required formulas are supplied.**

**The numerical results from the ANSYS ACP Workbench analysis program and the analytical results produced with the MATLAB application were compared. Different methods employed by other studies when modifying some design components such as modulus ratio and aspect ratio(a/b) have shown that the aspect ratio is inversely proportional to the buckling. Comparisons between various plate fiber directions, layer thicknesses, plate dimensions, and composite plates were made. Higher resistance to buckling and the largest buckling load are obtained by orienting the laminate at a 90° angle.**

***KEYWORDS:*** *Uniaxial Buckling, Boron Epoxy laminated composite, Critical buckling load.*

## 1. INTRODUCTION

The future of the material industry will be determined by how quickly traditional construction materials are replaced by composite materials. Composite laminates have significantly different resistance to load applied normal to the fiber orientation than load applied in the fibers' direction[1]. Many parameters influence the mechanical properties of composite laminates, including fiber type, matrix type, fiber orientation, ply stacking sequence, laminate thickness, aspect ratio, boundary conditions, delamination size, and temperature[2]. Buckling failure is one of the most dangerous failure types for composite laminates. Buckling can happen at loads exceeding the strength limit, and it causes significant structural deformation[3]. Fiber orientation, ply stacking sequence, from boundary conditions, aspect ratio, laminate thickness, stiffening type, loading situation, and the number of layers of the laminate are only a few of the variables that affect the critical buckling loads of composite laminates[4]. According to Topal and Uzman, the number of layers in composite laminates gradually increases the maximum buckling load for symmetric laminates, whereas for asymmetric laminates, the maximum buckling load increases rapidly from two to four layers before slowing down[5]. Asymmetric rectangular laminates have a larger critical buckling stress than symmetric specimens, according to analytical and FEA studies[6]. Topal [7] looked at the ideal fiber orientation, aspect ratio, and boundary conditions for a six-layer laminate that was subjected to a biaxial load and had simple supported boundary circumstances. The optimum fiber orientation of four-ply simply supported rectangular laminate plate with aspect ratio a/b = 3 under uniaxial compression load is $[45/-45]s^{\circ}$[8]. Shufrin, et al., 2008 [9] presented the buckling analysis of symmetric cross and angle ply under uniaxial and biaxial compression load laminated rectangular composite plates with various edge conditions.

In current study, The Classical Laminated Plate Theory (CLPT) is used to investigate the critical buckling of uniaxial for angle and cross-laminated composite plate under different

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orientation angle of lamina.

## 2. BUCKLING OF COMPOSITE THIN PLATES

Composite materials are created by combining two or more materials with varying mechanical characteristics. Laminated composite plates are commonly utilized in engineering projects such as ships and airplanes. This article examines the buckling of composite plates used as components of structure.. Consider the composite plate in figure 1. with dimensions of length *a*, length *b*, and height *h* [10] [11].

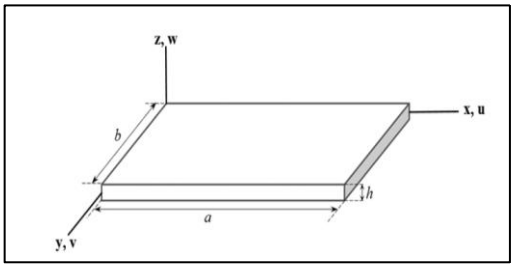


**Fig.(1):-.** Coordinate system and geometry of plate

The boundary conditions for all sides simply supported plate (SSSS) are as follows:

$$
\begin{aligned}
&x=0; && w(0,y)=0; && M_x\,(0,y)=0\\
&x=a; && w(a,y)=0; && M_x=(a,y)=0\\
&y=0; && w(x,0)=0; && M_y=(x,0)=0\\
&y=b; && w(x,b)=0; && M_y=(x,b)=0
\end{aligned}
\qquad (1)
$$

where $M_x$, $M_y$ are the bending moments as shown in Figure 2. and *w* is the out-of-plane displacement of the plate [11]

$$M_x = -\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -\left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2}\right) \qquad (2)$$

$$M_{xy} = -2\,D_{66}\frac{\partial^2 w}{\partial x\,\partial y}$$

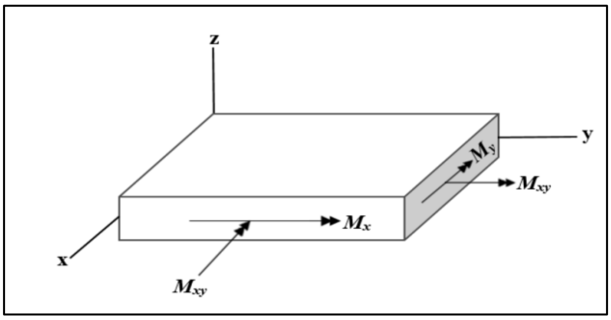


**Fig.(2):-** Moment resultants of thin plate

### 2.1 Governing Equations of Buckling of Rectangular Composite under Uniaxial Compression

The Classical Lamination Theory, which is practically identical to the Classical Plate Theory with the exception of Anisotropy, is used to analyze laminated plates. The laminate must be thin in cases when the distance between '*a*' and '*b*' is higher than *10* times the thickness '*t*'. It also calls for a little displacement in the transverse direction *'w'*, where *w* is much smaller than '*t* .The Kirchhoff's assumptions are used to build the Classical Plate Theory (CPT) [12]. The buckling load of laminated composite plates is more complicated to calculate than CPT. Kirchhoff's assumptions for orthotropic and composite multilayer thin plates have been expanded. By creating CPT, the Classical Laminated Plate Theory (CLPT) for composite plates was established. For symmetrically laminated cross ply plates, there is no coupling between bending and twist, and $D_{16}$, $D_{26}$ are equal to zero. The Classical Laminated Plate Theory CLPT has the following form[10] [11]".

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} + N_y\frac{\partial^2 w}{\partial y^2} \quad (3)$$

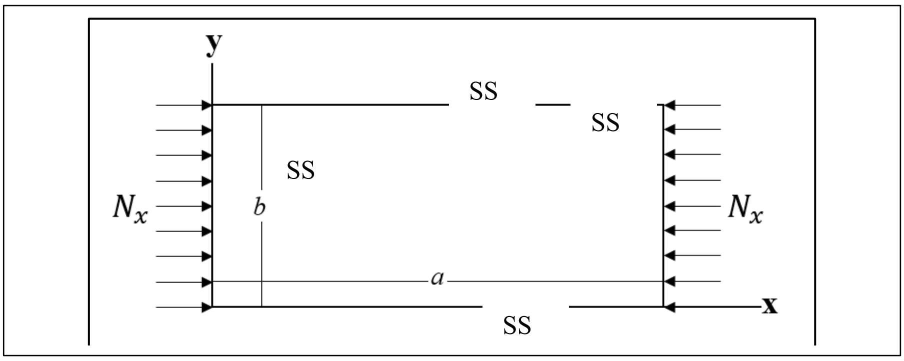


**Fig.(3):-.** The boundary condition of composite plate, (simply supported on all four sides).

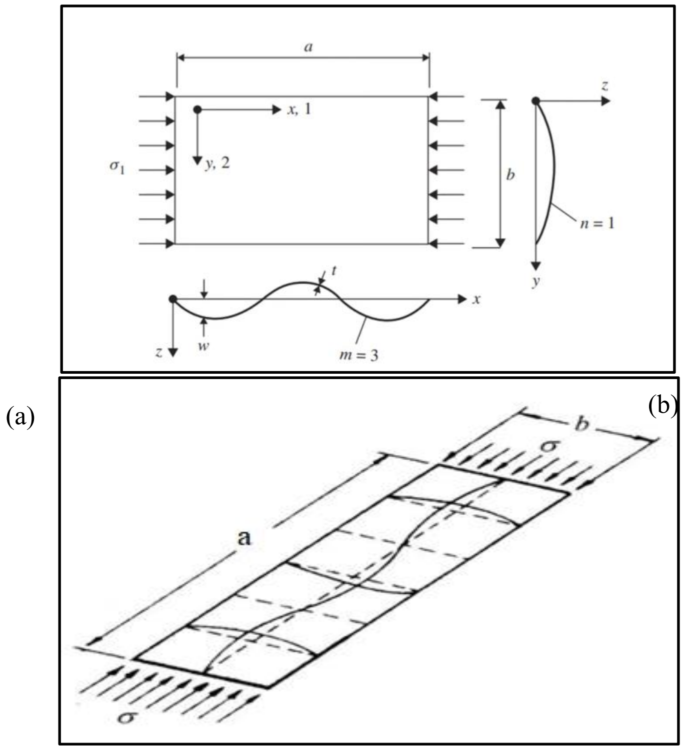


**Fig.(4):-** ( a and b) Shape of m, n (m=3,n=1)

According to Eq. 3, as illustrated in Figure 3, when the plate is only subjected to compression load $N_x$ in the $x$ direction, $N_{xy}= N_y = 0$. Assume that the compression loads per unit width are constant.

By assuming the buckling mode forms, the boundary requirements given by Eq. 1 are perfectly solved.

$$w_{mn} = C_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad m,n = 1,2,3 \ldots.. \tag{4}$$

" $C_{mn}$ is a small arbitrary amplitude coefficient, and *m,n* are the number of half-waves in the x and y directions, as shown in Fig.4. When boundary conditions for all sides simply supported plate are substituted in Eq. 3, the following critical load is obtained [10]";

$$(N_x)_{cr} = \pi^2 \left[D_{11} \left(\frac{m}{a}\right)^2 + 2(D_{12} + 2D_{66})\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4 \left(\frac{a}{m}\right)^2\right] \tag{5}$$

Where

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (Q_{ij})(z_k)^3 - (z_{k-1})^3 = \sum_{k=1}^{N} (Q_{ij})\left(t_k (z_k)^2 + \frac{(z_k)^3}{12}\right) \quad i,j = 1,2,3 \tag{6}$$

"After the uniaxial compression loading, the critical load was obtained in Eq. 5 for the composite plate. The minimum $(N_x)_{cr}$ is obtained when $n$ equals one. From CPT matrix can be expressed Eq. 7 using non-dimensional parameters[10]",

$$N = \frac{N_x b^2}{D_{22}\pi^2} = \left[\frac{D_{11}}{D_{22}} \left(\frac{b}{a}\right)^2 m^2 + 2\left(\frac{D_{12}}{D_{22}} + 2\frac{D_{66}}{D_{22}}\right) + \left(\frac{a}{b}\right)^2 \frac{1}{m}\right] \tag{7}$$

Where $N$: Non-dimensional buckling load

**2.2.** Macromechanical Properties of Composite Materials

"Engineering constant relationships for orthotropic materials are more complicated. Only the necessary formulas are provided below, and the MATLAB code written using classic laminate plate theory.;[10]".

"The unidirectional reinforced lamina in the 1-2 plane is shown in Figure 5. . After the necessary arrangements are made, the stress strain relationship is obtained as follows[10]"

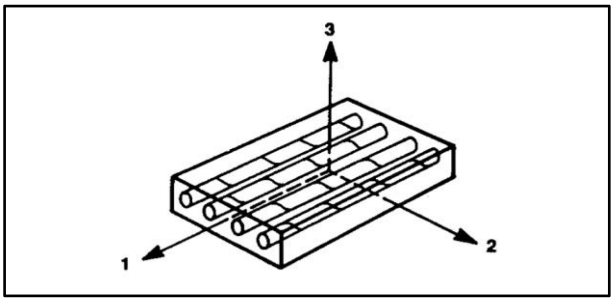


**Fig.(5):-** Unidirectional Reinforced Lamina [10]

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{pmatrix} = \begin{pmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{pmatrix}\begin{pmatrix}\sigma_1\\ \sigma_2\\ \gamma_{12}\end{pmatrix} \tag{8}$$

"For the orthotropic lamina, the $Q_{ij}$ in terms of engineering constants is given below [10]"

$$Q_{11} = \frac{E_1}{1 - \upsilon_{12}\ \upsilon_{21}}$$
$$Q_{22} = \frac{E_2}{1-\upsilon_{12}\ \upsilon_{21}} \tag{9}$$

$$Q_{12} = \frac{\upsilon_{12}\ E_2}{1-\upsilon_{12}\ \upsilon_{21}} = Q_{21}$$
$$Q_{66} = G_{12}$$

"The stresses and strains relations for an orthotropic material were defined in the principal material coordinates. Because laminae might be oriented differently, as shown in Figure 6, a relationship between stresses and strains in the major material coordinates and those in the body coordinates is required. Where $E_1$ and $E_2$ are Young Moduli, $G_{12}$ is shear modulus, and $\upsilon_{12}$ and $\upsilon_{21}$ are Poisson ratios (see Figure 5 for the direction of number (1) to the x-axis). [10]"

The relationship between $\upsilon_{12}$ and $\upsilon_{21}$ is as follows [10]

$$\frac{\upsilon_{12}}{E_1} = \frac{\upsilon_{21}}{E_2} \tag{10}$$

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{pmatrix} = \begin{pmatrix}\overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}}\\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}}\\ \overline{Q_{16}} & \overline{Q_{16}} & \overline{Q_{66}}\end{pmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix} \tag{11}$$

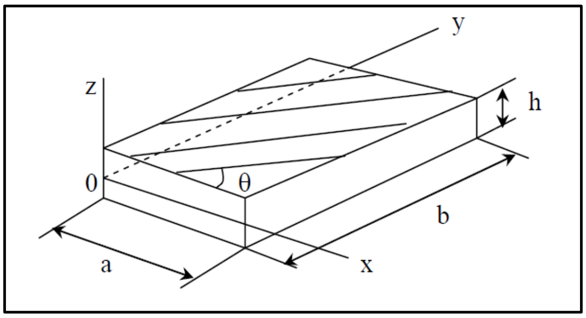


**Fig.(6):-** Fiber orientation in x-y coordinates [10]

where $\overline{Q_{ij}}$ are transformed reduced stiffness matrix elements, the terms of this matrix are obtained as follows

$$\begin{aligned}\overline{Q_{11}} &= Q_{11}\ cos^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta\ cos^2\theta + Q_{22}\ sin^4\theta\\ \overline{Q_{12}} &= (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta\ cos^2\theta + Q_{12}\ (sin^4\theta + cos^4\theta)\\ \overline{Q_{22}} &= Q_{11}\ sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta\ cos^2\theta + Q_{22}\ cos^4\theta\\ \overline{Q_{66}} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta\ cos^2\theta + Q_{66}\ (sin^4\theta + cos^4\theta)\end{aligned} \tag{12}$$

The lower and upper limit of ply $z_{(k-1)}$ and $z_{(k)}$ shown in Figure 7., respectively. The "$z$" direction is chosen downward positive and is chosen as the starting point of the mid-plane. N represents the total number of layers.

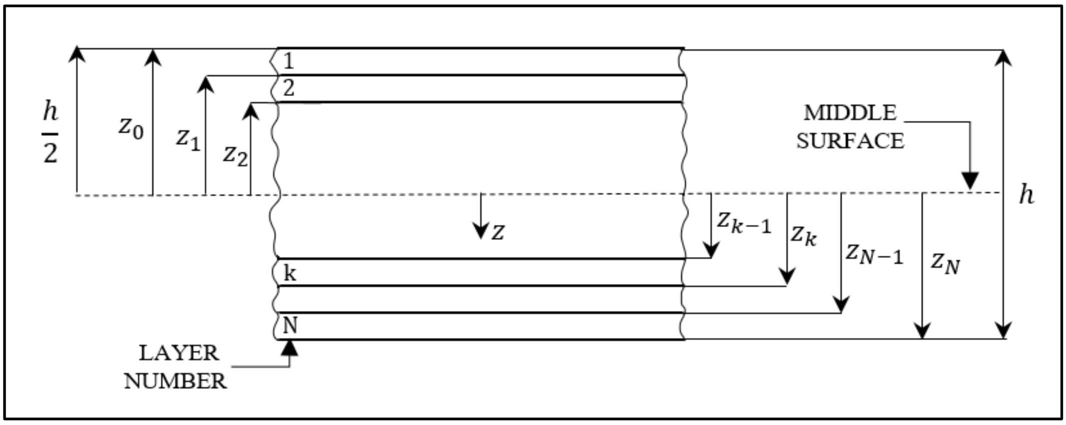


**Fig.(7):-**. Geometry of an N-layered laminate

## 2.3 MATERIALS AND PROPERTIES

Properties of two kind of lamina indicated in Table (1).

**Table (1)** Typical Mechanical Properties of a Unidirectional Lamina[10]

| Property | Symbol | Units | Glass/Epoxy | Boron/Epoxy |
|---|---|---|---|---|
| Fiber volume fraction | V1 | | 0.45 | 0.5 |
| Longitudinal elastic modulus | E1 | GPa | 38.6 | 204 |
| Transverse elastic modulus | E2 | GPa | 8.27 | 18.5 |
| Longitudinal/Transverse modulus | E1 / E2 | | 5 | 10 |
| Major poisson's ratio | $\upsilon_{12}$ | | 0.26 | 0.23 |
| Shear modulus | G12 | GPa | 4.14 | 5.59 |

## 3. MODELLING IN ANSYS ACP

To predict the Critical buckling load must be created first using ANSYS Composite PrepPost is a specialized tool for sample preparation and evaluation of composite structures results. Program ANSYS Composite PrepPost is fully integrated in ANSYS Workbench platform[13].

- Start ANSYS workbench and drag ACP (Pre) to project schematic;
- Add the materials (Boron-epoxy) to engineering data;
- Enter required material as shown Table (1) from engineering data;
- Enter the desired rectangle (plate 500mm X 250mm X 3mm) in the geometry and design and select the edit surface to adjust the thickness to 0 mm so that Modal can be opened in the next step;
- Open model cell, assign material (Boron-epoxy) at 0.75 mm thickness and complete mesh as shown in Figure 8.
- Open setup, select and create Fabrics under Material Data (Boron-Epoxy) to prepare the stacks;
- Create Ply with different angles as required; [0,0] or [30,30]
- Use oriented elements to create a Ply Group and enter angles 0 and 0, repeat the process for 2 layers;
- Add new rosette to model and define a rosette type as parallel;
- Create new oriented element set using rosette and elements set as required;
- Update the model and convert to a solid model;
- Drag and drop static structural workbench to project schematic and connect setup cell from ACP (Pre) to model cell in modal select option "transfer solid data"; then drag with Eigenvalue Buckling as shown in Figure 8.
- Open the model cell in Modal and Check the geometry; then applied mesh as shown in Figure 10 and the boundary condition as shown in Figure 9 and also indicated in Eq. 1. and running the study;

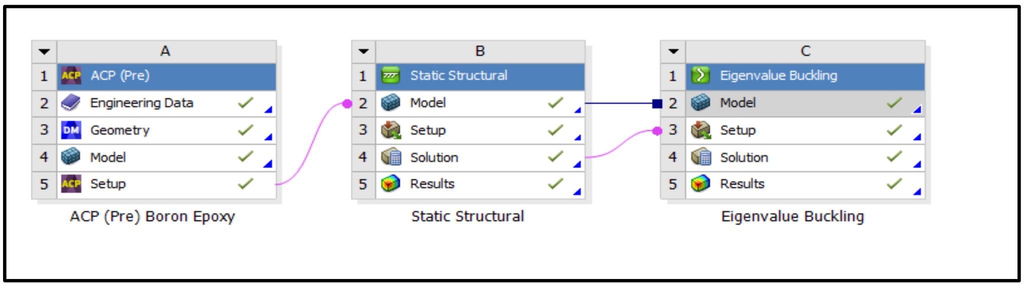


**Fig.(9):-** ANSYS ACP (Pre) link with Static Structural and Eigenvalue Buckling(project schematic)

Rectangular composite plates are simply supported on all four sides. Each node has six degrees of freedom, which are indicated by the letters UX, UY, UZ, ROTX, ROTY, and ROTZ, where ROT corresponds to rotational degrees of freedom.

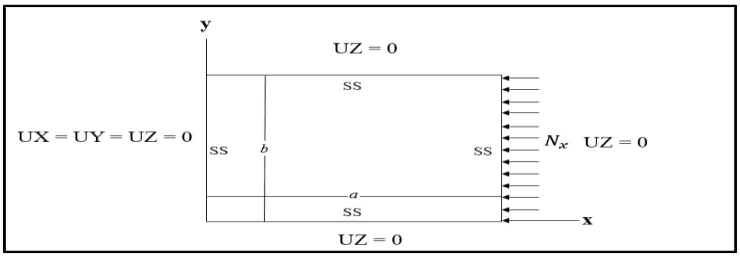


**Fig.(9):-** Boundary Conditions

Static solution is carried out under uniaxial compression force to determine the stiffening of the structure. Although there are a few buckling modes, there are theoretically an endless amount of buckling modes. The first buckling mode calls for the least amount of force. The user enters this mode number during eigenvalue buckling solution. For each mode, buckling loads are now computed. The values that were provided for the study are those that apply when m = n = 1.
The MATLAB code generated with the formulas described in section 2 was used to derive the analytical solution[14]. The project schematic of ANSYS Workbench 2020 R1, which is depicted in Figure 8, was utilized to get to at the numerical solution.

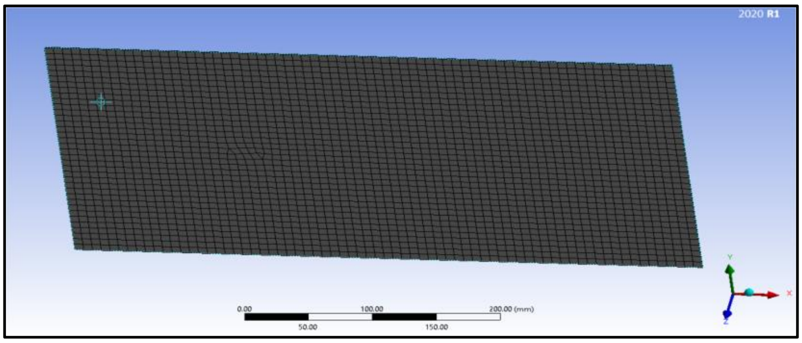

**Fig.(10):-** meshed of plate a/b=2 ratio

## 4. RESULTS AND DISCUSSIONS

### 4.1 Number of Layers

Critical loads were determined for plates with dimensions of 500 mm by 250 mm with varied fiber orientations and layer thicknesses. Each layer is assumed to be 0.75 mm thick (tk), resulting in a total plate thickness of 1.5 mm for a 2-layer plate and 3 mm for a 4-layer plate made of boron/epoxy material. In Table 2, comparisons are presented.

**Table( 2):-** Critical load for different number of layer and fiber orientation

| θ (degree) | Number of Layer | | | | | |
|---|---|---|---|---|---|---|
| | 2 | | | 4 | | |
| | MATLAB N/mm | ANSYS N/mm | Error(%) | MATLAB N/mm | ANSYS N/mm | Error (%) |
| 0 | 7.0875 | 6.901 | 2.63 | 56.6997 | 55.77 | 1.63 |
| 30 | 17.6550 | 16.98 | 3.82 | 141.2400 | 140.45 | 0.55 |
| 45 | 26.3690 | 25.87 | 1.89 | 210.9524 | 208.95 | 0.94 |
| 60 | 33.2296 | 32.76 | 1.41 | 265.8367 | 264.22 | 0.60 |
| 90 | 38.2366 | 37.64 | 1.56 | 305.8931 | 303.12 | 1.34 |

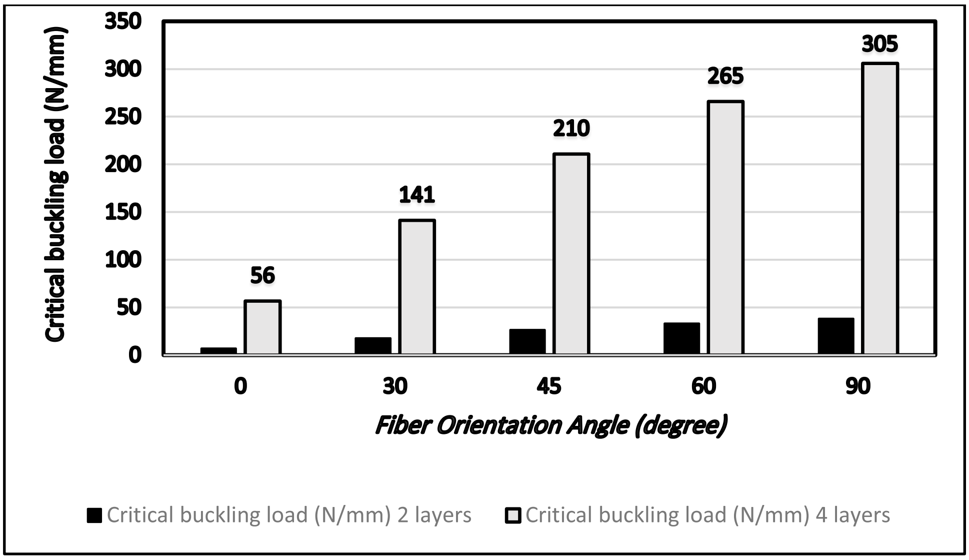


**Fig.(11):-** Critical buckling load for 2and 4 number of layer and different fiber orientation

The critical load clearly increases as the number of layers increases. Likewise, when the fiber orientation angle increases, it also raises the critical load. When comparing the numerical and analytical solutions. The critical load of the composite plate with 4 layers is almost seven times higher than the critical load of the composite plate with 2 layers as shown in Figure 11. It is clear that the critical load increases as the number of layers increases. Similarly, as the fiber orientation angle increases, the critical load increases.

## 4.2 Fiber Orientations

Critical buckling loads were calculated in different fiber orientation for plate with dimensions of 500 mm x 250 mm. The thickness of plate is taken as 3 mm ( h). Comparisons are shown in Figure 12, that the critical load rises with increasing fiber orientation angle. Additionally, several fiber orientation angles have been used, and in each case, the thickness remains the same. Several variations of fiber orientation angles have been applied and the thickness is the same under all conditions. The highest buckling load is obtained when the 90° fiber orientation angle is outer layer according to the mid-plane.

The Figure 12 shows that when the outer layer is kept constant whilst the fiber orientation of the inner layer varied from 0º- 90º, the critical buckling load does not change much. However, if the inner layer is kept constant whilst the fiber orientation of the outer layer varied from 0º- 90º, the critical buckling load decreases rapidly until θ value of 50º, after that the critical buckling load is almost constant. This indicates that the orientation of the outer layer plays more significant role on the load carrying capability of the laminate. For more detailed the

[90/ 90] and [90/ 45/45/90] laminates are the best optimization for Critical buckling loads among all different 16 laminates.

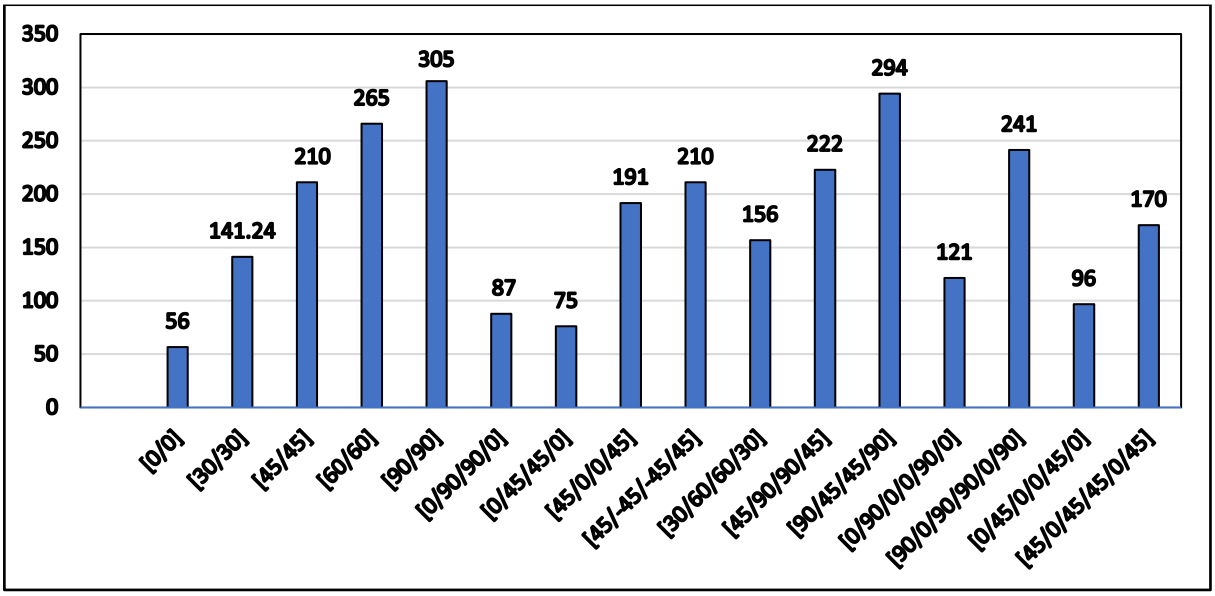


**Fig.(12):-.** Critical buckling load for different fiber orientation with dimensions (500 X 250 X 3) mm

**4.3 Effect of a/b Ratio**

For the plate with fiber orientation 0/90/90/0 and varied a/b ratios, critical buckling loads were computed. Plate length is assumed to be 500 mm (a), and plate thickness is assumed to be 3 mm (h). In Figure 13, comparisons are displayed. As the a / b ratio increases, the critical buckling load increases.

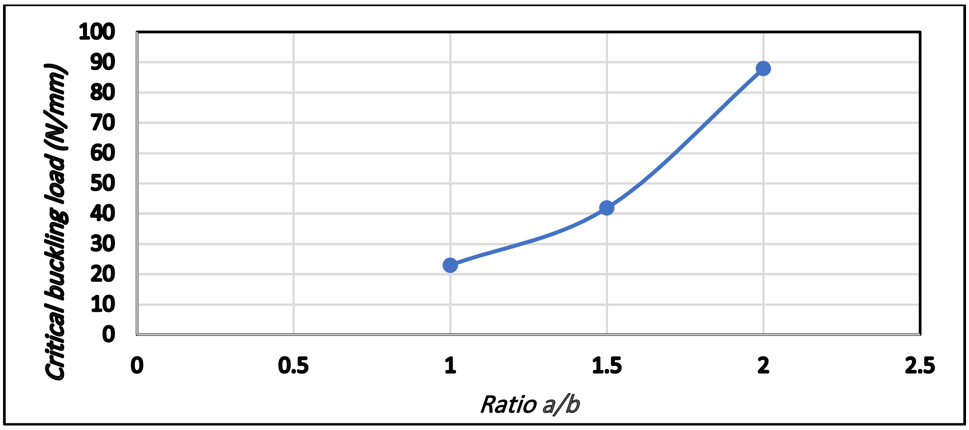


**Fig.(13):-** Critical buckling load with Ratio a/b

**4.4 Non-dimensional buckling load**

The results for the composite plate with various edge conditions showed, stacking sequence, (a\b) and modulus ratio giving a good agreement when compared with Reddy, 2003 and Dr.Widad., 2019. Table 3 demonstrates that the Non-dimensional buckling load ($\bar{N}$) behavior of laminated plates is consistent with that seen by other studies for various aspect ratios and modulus ratios ($E_1/E_2$=5 and $E_1/E_2$=10).

**Table (3):** -Non-dimensional buckling load ($\overline{N}$) , for [0 90 90 0] plates (simply supported on all four sides) of different aspect, modulus ratio, by using Eq.7

| References | $\frac{a}{b}$ | $\frac{E1}{E2}=5$ Glass/Epoxy | $\frac{E1}{E2}=10$ Boron/Epoxy |
|---|---|---|---|
| Present work | 0.5 | 13.5 | 18.39 |
| Dr.Widad 2019 [15] | | 13.94 | 18.22 |
| Reddy, 2003 [16] | | 13.9 | 18.13 |
| Present work | 1 | 5.59 | 6.13 |
| Dr.Widad 2019 [15] | | 5.66 | 6.35 |
| Reddy, 2003 [16] | | 5.65 | 6.34 |
| Present work | 1.5 | 5.24 | 4.97 |
| Dr.Widad 2019 [15] | | 5.23 | 5.28 |
| Reddy, 2003 [16] | | 5.23 | 5.27 |

The deformed form of the plates that has a/b=2 in different mode is shown in Figure 15. The first mode gives the smallest critical buckling load Ncrx= 6.9 N/mm.

After initiation of the dynamic analysis, the plate moves to the nearest stable equilibrium configuration, which is represented by center of plate. This configuration is shown in Figure 16 and as can be seen, the plate has an out-of-plane deflection with a half-wave in the x- and y directions (m=1,n=1).

As the edge displacement values increase of plate, the plate continues to deform statically out-of-plane with out-of-plane deflection represented by Figure 17 m=2,n=1 and Figure 18 m=3,n=1 . Depends the equation 4 as $m, n$ are the number of half-waves in the $x$ and $y$ directions.

After the verification of finite element code different analysis were performed for different stacking sequences and aspect ratio in which the number of layers and thickness of each layer are kept constant. It is found that the critical buckling load is increasing with the decreasing of aspect ratio under the Uniaxial compressive loading. Interestingly it is observed that the stacking sequence plays an important role in buckling load factor of the laminate.

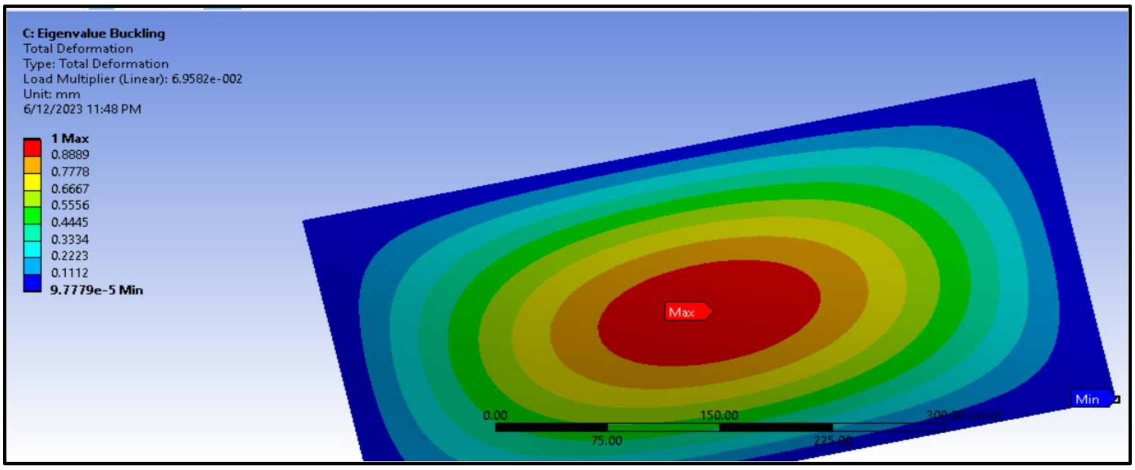


**Fig.(15):-** Calculation of (Nx)cr= 6.9 N/mm the first mode m=1,n=1 at stack equal [0 0] ANSYS ACP

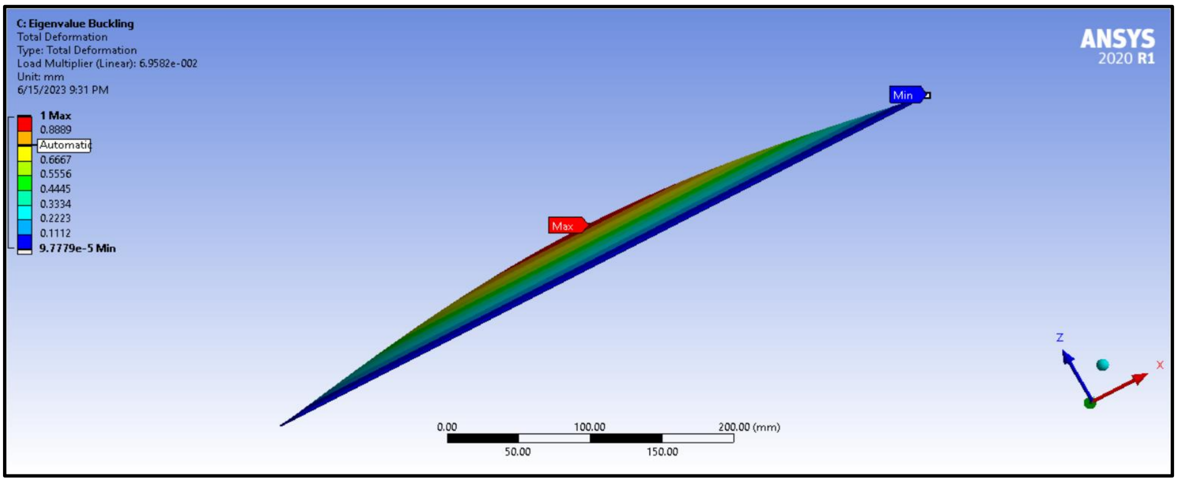


**Fig.(16):-.** First mode shape of laminated plate m=1, n=1 at stack equal [0 0]

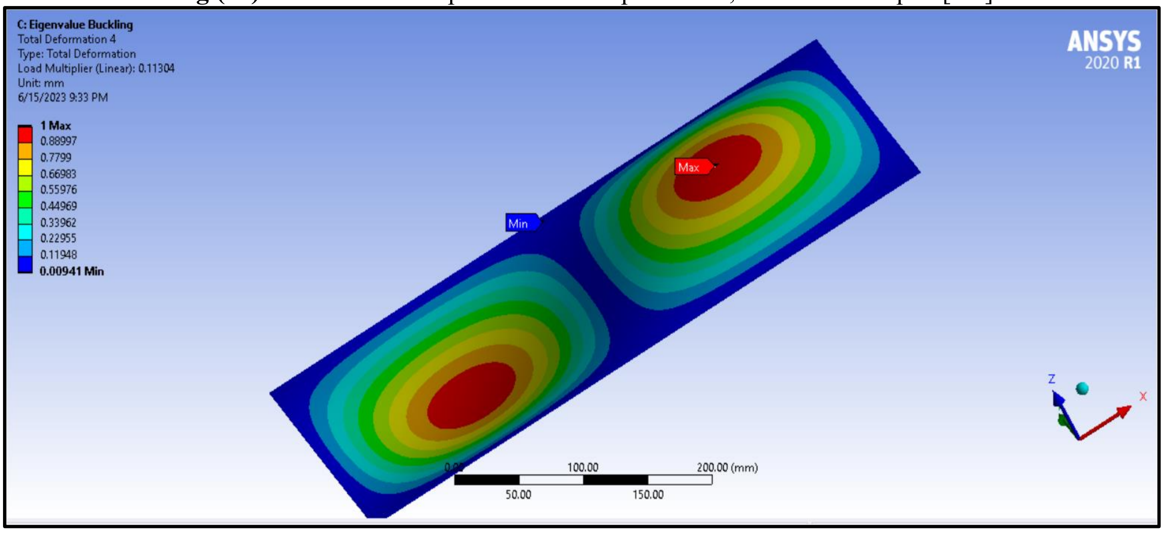


**Fig.(17):-** Second mode shape of laminated plate, m=2,n=1, stack equal [0 0]

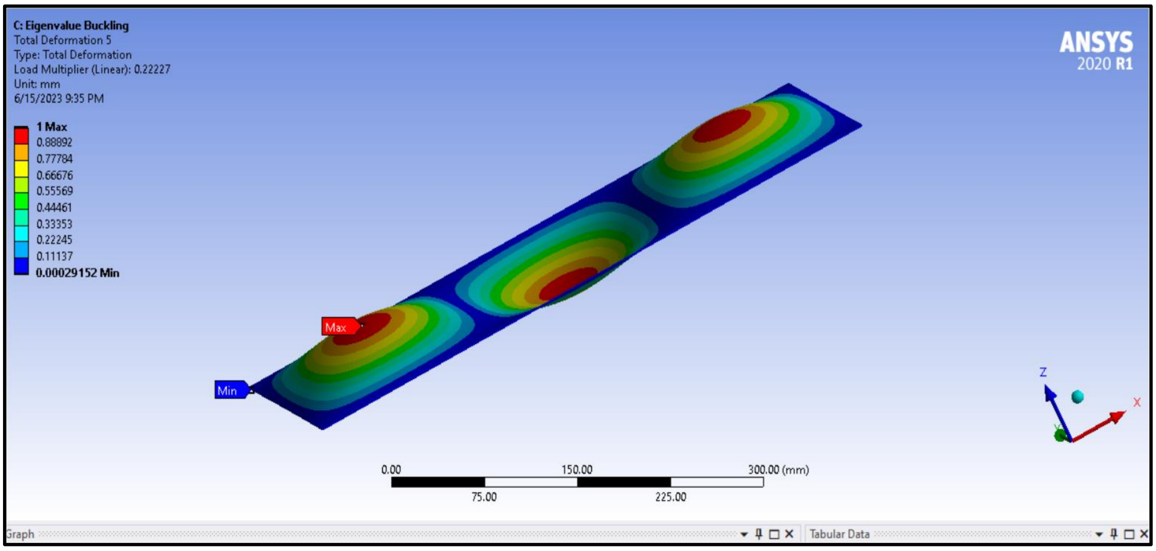


**Fig.(18):-** Third mode shape of laminated plate m=3, n=1 at stack equal [0 0]

## 5. CONCLUSION

The buckling load under the compressive load of composite thin rectangular plates was calculated in this work using MATLAB for analytical solution and ANSYS ACP 2020 R1 for numerical solution. The findings from the two programs were then compared. In essence, four cases were compared.

**1.** In the first illustration, given that one layer thickness was the same, the behavior of plates with the same dimensions in various fiber orientations for the 2-layer and 4-layer plate was explored.

**2.** The critical buckling load rises as the fiber orientation angle rises.

**3.** It is evident that the critical buckling load significantly increases as the number of layers is raised from 2 to 4.

**4.** It can be said that increasing the number of layers and changing the fiber orientation angle from 0° to 90° degrees that increases the stiffness of the plate. When comparing the numerical solution and the analytical solution.

**5.** The aspect ratio is inversely proportional with the buckling as proved by other methods used in other researchers when changing some design parameters such as modulus ratio and aspect ratio ($a/b$). The ratio between the edges of plates with the same fiber orientation angle and the same thickness has been changed and loading was applied from short edge. According to the data obtained as a result of the applications, the critical buckling load increases as the ratio of the edge length increases.

**6.** The higher buckling load is achieved at 90°, and when comparing the 16 variations, the greater the outermost layer angle in relation to the middle plane, the more resistant the plate is to buckling.

**Nomenclature**

| Symbols | Description | Units |
|---|---|---|
| a, b | plate length and width, respectively | mm |
| E1,E2, E3 | modulus of elasticity in 1, 2, and 3 directions ,respectively | GPa |
| G12, G23, G13 | shear modulus in plane 1-2, 2-3, 1-3 , respectively | GPa |
| h | thickness of the laminate | mm |
| zk, z(k-1) | distances from the reference plane of the laminate to the two surfaces of the kth ply | mm |
| tk | thickness of layer and is the distance from the mid-plane to the centroid of the layer | mm |
| k | layer number | ------- |
| L | total number of layers in the laminate | ------- |
| Mx, My, Mxy | bending and twist moments per unit length acting on a laminate | N.m/m |
| u, v, w | displacements in x, y, z directions, respectively | mm |
| $w(x,y)$ | flexural displacement of a plate | mm |
| (Nx)cr | critical buckling load | |
| $D_{ij}$ | bending stiffness matrix | |
| $\bar{Q}_{ij}$ | transformed reduced stiffness | |

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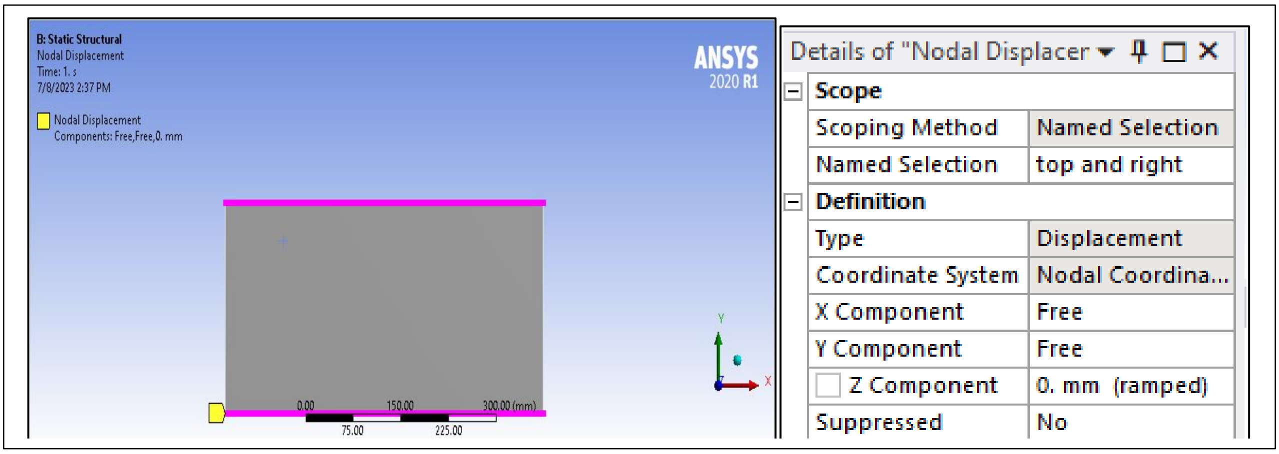
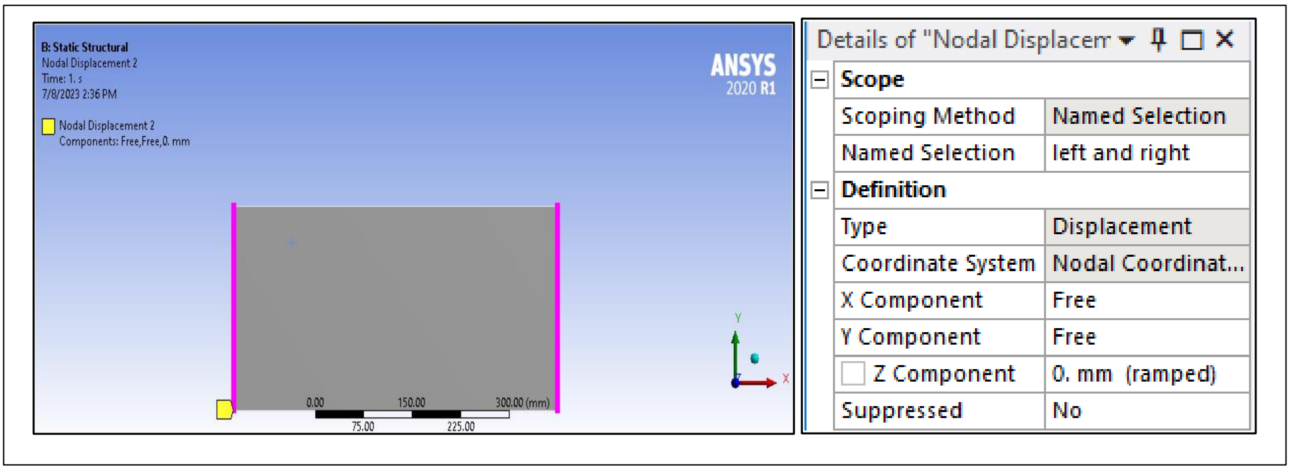
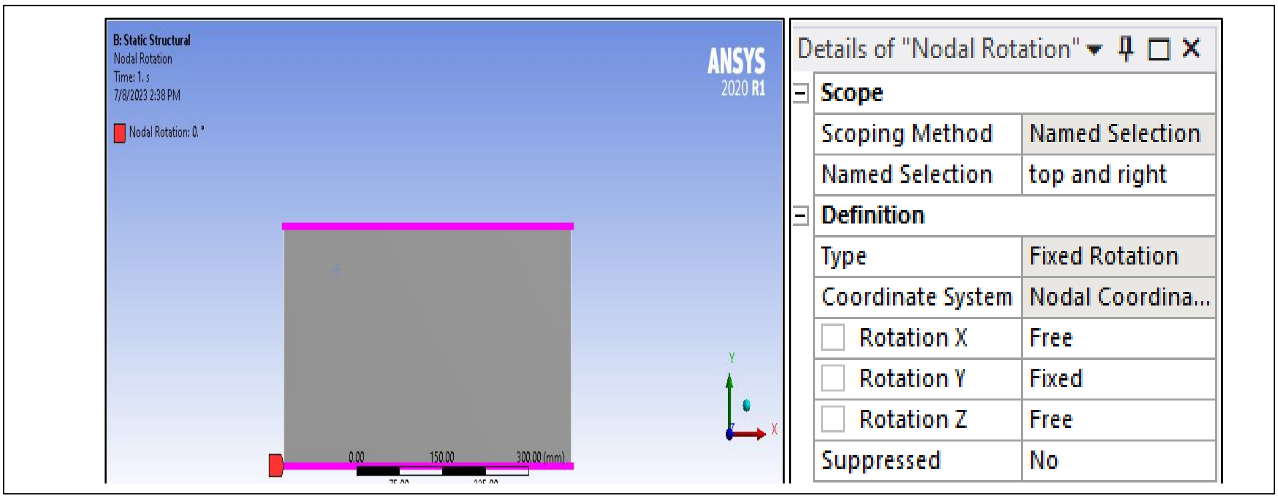
## Appendixes A



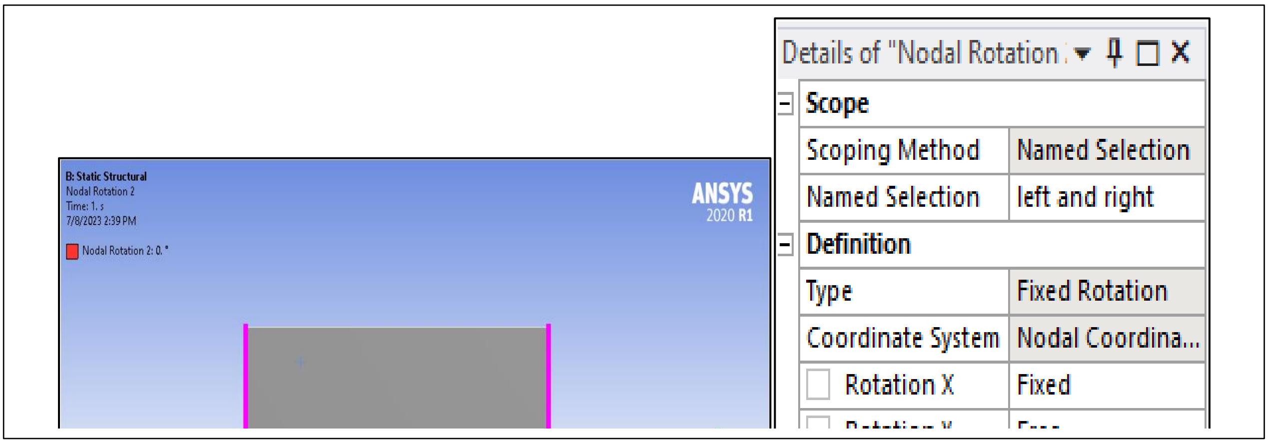

**A 1.** Boundary condition of Nodal Displacement of Top and Bottom side



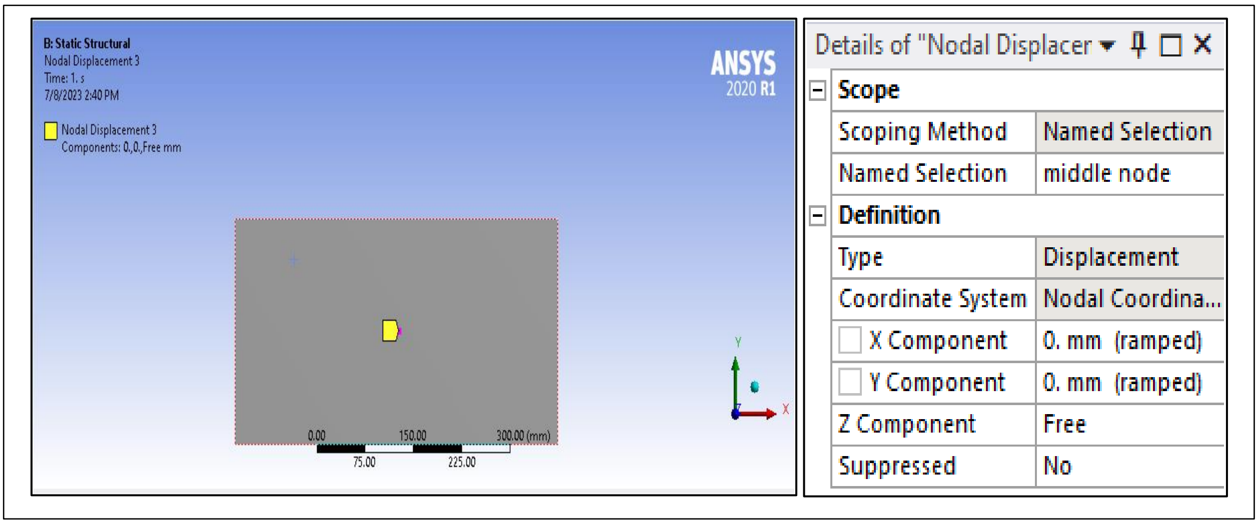

**A 2**. Boundary condition of Nodal Displacement of lift and right sid



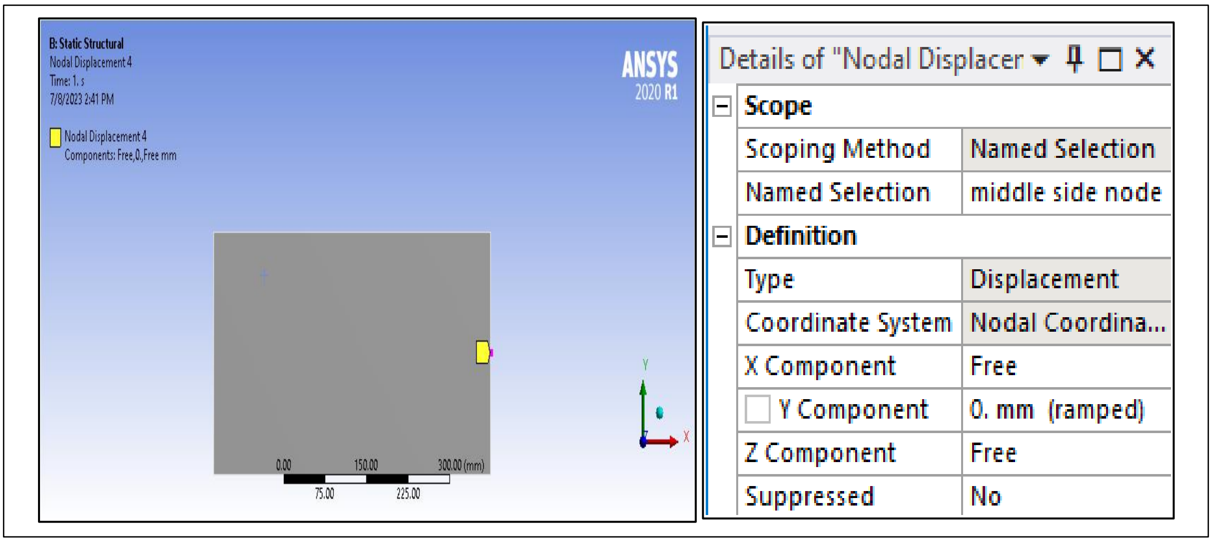

**A 3.** Boundary condition of Nodal Rotation of top and bottom side



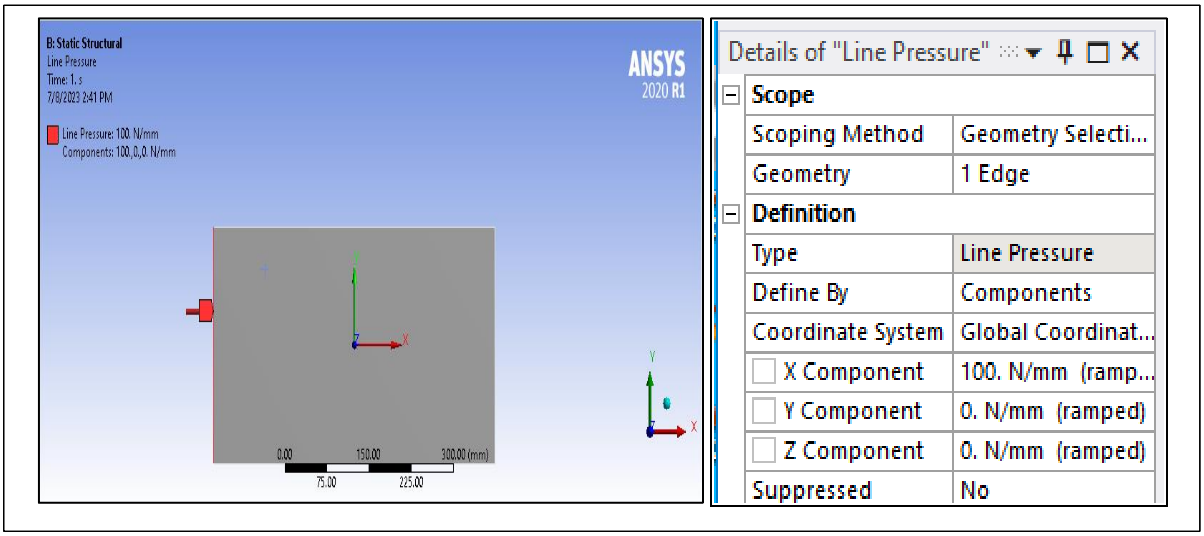

**A 4.** Boundary condition of Nodal Rotation of lift and right side



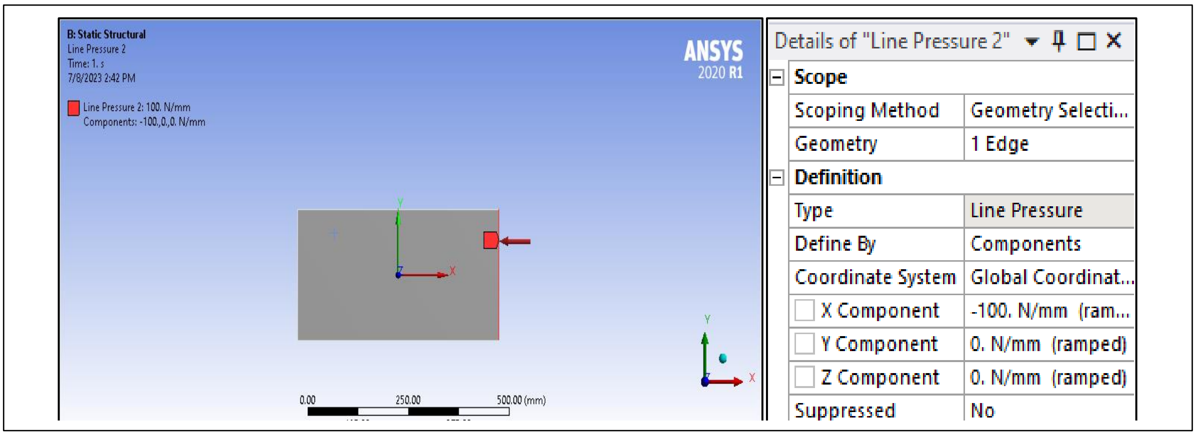

**A 5.** Boundary condition of Displacement of middle node



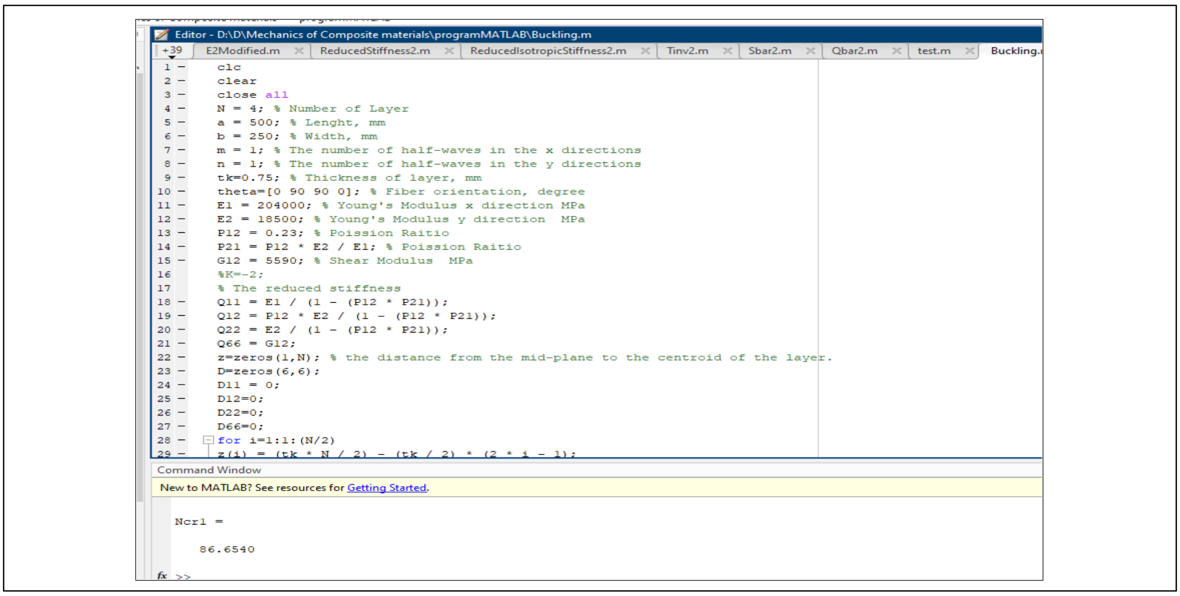

**A 6.** Boundary condition of Displacement of middle side node




**A 7.** Boundary condition of Pressure at lift side




**A 8.** Boundary condition of Pressure at right side




**A 9.** Result of ((*Nx)cr*=86.65 N/mm) MATLAB of Boron/Epoxy [0 90 90 0] with dimension 500 X 250 X 3 mm