

ENHANCING STAND-ALONE GPS CODE POSITIONING USING STAND-ALONE DOUBLE DIFFERENCING CARRIER PHASE RELATIVE POSITIONING

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ABSTRACT

Pseudo-range GPS code observables can provide absolute stand-alone positioning with accuracy of a few meters, which may not be suitable for a wide range of engineering applications. Differencing GPS observations (DGPS) can be used for reducing or removing some of GPS errors based on the high correlation between these errors over short baselines providing accurate relative positioning. Stand-Alone Double Differences Carrier Phase (SADDCP) is an accurate velocity estimation method based on single frequency stand-alone GPS observables. Precise GPS relative positioning can then be achieved by integrating the velocity over epoch. SADDCP is a double differences technique including two epochs, two satellites and one receiver. In SADDCP, the ambiguity and receiver clock errors are removed, whereas satellite clock error, orbit errors, and ionospheric and tropospheric delays are reduced significantly. Multipath remains and can be reduced based on the multipath correlation over time, and receiver noise is increased.

In this paper, SADDCP will be used to enhance the performance of stand-alone GPS code positioning, where the two positioning techniques are integrated using Kalman filter. The precise relative positioning provided by SADDCP will be utilized to smooth the absolute low accurate stand-alone GPS code positioning, providing enhanced absolute single frequency stand-alone GPS positioning. Tests in different GPS environments will be carried out for reliable investigations and the results will be discussed in details showing the advantages and limitations of this integration.

KEYWORDS: GPS, Code positioning, Single frequency, Double differencing, Kalman filter, Relative positioning.

1. INTRODUCTION

The GPS pseudo-range between receiver and satellite is obtained by matching the satellite code with the internal code generated by the receiver and scaling the time difference by the speed of light. Pseudo-range GPS code observables can provide absolute stand-alone positioning with accuracy of a few meters which may not be suitable for a wide range of engineering applications, such as mapping, cadastral surveying, geodetic control, and structures deformation. This is attributable to the pseudo-range error sources, such as satellite errors (clock and orbit), propagation errors (ionosphere, troposphere, and multipath), and receiver errors (clock, measurements noise, and phase center variation) [1].

The carrier phase observation is formed by stripping the code from the received signal. Carrier phase observation can be measured to the level of 0.01 cycles giving millimeters accuracy. Just the fractional phase with the accumulated integer number of wavelengths can be measured

by the receiver as the connection between the satellite and receiver is available. As for the initial total number of integer wavelengths, it is unknown which makes the absolute stand-alone one epoch based positioning impossible for carrier observations. This initial unknown number is known as the integer ambiguity. Differencing GPS (DGPS) observations can be used for solving this problem providing precise relative positioning. Relative positioning aims at determining the coordinates of an unknown point with respect to a known point or determining the vector between the two points (baseline) and this requires simultaneous observations at the two points. With DGPS, some of GPS errors are reduced or removed based on the high correlation between these errors over short baselines. Differencing observations can be formed using code or carrier phase measurements taking one of the following forms: single, double and triple differences. Single differences can be formed between two receivers, two satellites, or two epochs. Double differences are formed between any two single differences, whereas triple differences are between the three

forms of single differences, including two receivers, two satellites, and two epochs [2].

Stand-Alone Double Differences Carrier Phase (SADDCP) is one of the differencing observations forms including two single differences between two satellites and two epochs with one receiver. In this double differences technique, single frequency carrier phase observables measured by the same receiver are firstly differenced cross epochs and secondly cross satellites. These two differences lead the ambiguity to be removed as long as the connection between the satellite and receiver is continues. Furthermore, receiver clock error is removed, satellite clock error is reduced based on the stability of the satellite clock over transmission times, satellite orbit errors are reduced, ionospheric and tropospheric delays are reduced to the change across the interval, multipath remains and can be reduced based on the multipath correlation over time, and receiver measurements noise is increased. SADDCP is

considered as an accurate velocity estimation method based on single frequency stand-alone GPS observables. Precise GPS relative positioning can then be achieved by integrating the velocity over epoch [3] [4] [5].

In this paper, SADDCP is used to enhance the performance of stand-alone GPS code positioning, where the two positioning techniques are integrated using Conventional Kalman Filter (CKF). The precise relative positioning provided by SADDCP is utilized to smooth and enhance absolute stand-alone GPS code positioning. This can help to gather the advantages of the two positioning techniques providing accurate absolute single frequency stand-alone positioning.

2. THE INTEGRATION MATHEMATICAL DESCRIPTION

2.1 Saddep Relative Positioning

The GPS carrier phase observable in meters can be written as [6]:

$$CP_{(s,r)(k)} = p_{(s,r)(k)} + c(dT_{(s)(k)} - dt_{(r)(k)}) + dion_{(s,r)(k)} + dtrop_{(s,r)(k)} + dor_{(s)(k)} + E_{(s,r)(k)} + L*N$$

where,

$CP_{(s,r)(k)}$	the carrier phase observation (m)
$p_{(s,r)(k)}$	the true range between receiver (r) and satellite (s) at epoch (k)
c	the speed of light
$dT_{(s)(k)}$	the clock error of satellite (s) at epoch (k)
$dt_{(r)(k)}$	the clock error of receiver (r) at epoch (k)
$dion_{(s,r)(k)}$	the ionospheric delay error between receiver (r) and satellite (s) at epoch (k) (m)
$dtrop_{(s,r)(k)}$	the tropospheric delay error between receiver (r) and satellite (s) at epoch (k) (m)
$dor_{(s)(k)}$	the orbit error of satellite (s) at epoch (k) (m)
$E_{(s,r)(k)}$	the measurement noise including multipath between receiver (r) and satellite (s) at epoch (k)
L	the carrier wavelength (m)
N	the unknown integer ambiguity (cycle)

The true range between receiver (r) and satellite (s) at epoch (k) can be written as:

$$p_{(s,r)(k)} = ((X_{(s)} - X_{(r)})^2 + (Y_{(s)} - Y_{(r)})^2 + (Z_{(s)} - Z_{(r)})^2)^{0.5}$$

where, X, Y and Z are the satellite and receiver Cartesian coordinates.

In SADDCP, the first single differencing is formed between one receiver (r), one satellite (s) and two adjacent epochs (k) & ($k + 1$). The single differencing equation can be written as:

$$SD_{(s)(r)(k+1,k)} = CP_{(s,r)(k+1)} - CP_{(s,r)(k)} = [p_{(s,r)(k+1)} + c(dT_{(s)(k+1)} - dt_{(r)(k+1)}) + dion_{(s,r)(k+1)} + dtrop_{(s,r)(k+1)} + dor_{(s)(k+1)} + E_{(s)(r)(k+1)} + LN] - [p_{(s,r)(k)} + c(dT_{(s)(k)} - dt_{(r)(k)}) + dion_{(s,r)(k)} + dtrop_{(s,r)(k)} + dor_{(s)(k)} + E_{(s)(r)(k)} + LN]$$

Where, $SD_{(s)(r)(k+1,k)}$ is single differencing between receiver (r), one satellite (s) and two adjacent epochs (k) & ($k + 1$)

From the single differences, the double difference ambiguity is removed as long as the integer ambiguity remains constant and the receiver keeps lock the satellite signal. Satellite clock error is reduced based on the stability of the

satellite clock over transmission times. Satellite orbit errors are reduced significantly based on the high correlation between the satellite orbit errors over time. Ionosphere and troposphere errors are reduced to the change in delay across the interval.

Multipath remains and can be reduced based on the multipath correlation over time. However, receiver clock error is doubled and receiver measurements noise increases. The final formula of single differences equation can be written as:

$$SD_{(s)(r)(k+1,k)} = p_{(s)(r)(k+1,k)} - c dt_{(r)(k+1,k)} + E_{(s)(r)(k+1,k)}$$

The second differencing in SADDCP is carried out between two single differences, similar to that in equation (3), cross two satellites (s) and (j). This can be written as:

$$DD_{(s,j)(r)(k+1,k)} = SD_{(s)(r)(k+1,k)} - SD_{(j)(r)(k+1,k)} = [p_{(s)(r)(k+1,k)} - c dt_{(r)(k+1,k)} + E_{(s)(r)(k+1,k)}] - [p_{(j)(r)(k+1,k)} - c dt_{(r)(k+1,k)} + E_{(j)(r)(k+1,k)}]$$

Where, $DD_{(s,j)(r)(k+1,k)}$ is double differences between one receiver (r), two epochs ($k+1$) & (k), and two satellites (s) & (j).

Receiver clock error is cancelled out in the double differences. This is extremely important for getting accurate results where the oscillators in low cost receivers vary in frequency with temperature and pressure making the receiver clock unreliable [6]. The final formula of double differences equation can be written as:

$$DD_{(s,j)(r)(k+1,k)} = P_{(s,j)(r)(k+1,k)} + E_{(s,j)(r)(k+1,k)}$$

The only unknowns in this equation are the receiver Cartesian coordinates in the two epochs (k) & ($k+1$). The changes in the receiver positions between the two epochs can be determined by fixing the coordinated of the receiver at epoch (k) (as zeros for example) and solving for the receiver coordinated at epoch ($k+1$). To determine the relative position of the receiver at epoch ($k+1$) from (k), the double differences equation should be written as:

$$b = A X + v$$

where, b the measurement vector with a size of (number of epochs -1, 1)

X the parameter vector with a size of (number of epochs * 3, 1) which include the change in Cartesian position across the interval

A matrix with a size of (number of epoch -1, number of epoch * 3) which relates the parameters to the states

v a vector of random noise with a size of (number of epochs -1, 1)

This equation can then be solved by least squares as:

$$X = (A^T w A)^{-1} A^T w b$$

Where, w is the weight matrix with a size of (number of epochs -1, number of epochs -1) which is based on the average satellite residuals

obtained from the stand alone code positioning calculations.

2.2 The Integration Of Sddcp/Code Positioning

Kalman filter (KF) filters measurements based on the expected changes of these measurements over time and the statistical properties of the system measurement errors. The filter determines the minimum error estimate of the states based on the linear relation between the measurements and these states. The states are composed of number of values that adequate to define the system motion [7]. KF consists of measurement model and dynamic model which will be illustrated here to define the basic elements as related to the integration of SADDCP with code positioning. As for the equations of propagation and update steps, they are well documented in different sources and there is no point for repeating here.

The measurement model defines the mathematical linear relationship between the measurements and the filter states. The discrete measurement model at the epoch (k) can be defined as:

$$Z_{(k)} = H U_{(k)} + v_{(k)}$$

where,

$Z_{(k)}$ the vector of measurements at the epoch (k)

$U_{(k)}$ the system state vector at the epoch (k)

H the design matrix measurement which defines the linear relationship between the states and the measurements

$v_{(k)}$ the measurement residual vector

The dynamic model describes the change in the state vector parameters over time. The discrete dynamic Model between epochs ($k+1$) & (k) can be given as:

$$U_{(k+1)} = M U_{(k)} + W_{(k)}$$

where,

M the state transition matrix that defines the relation between state vector parameters over time.

$W_{(k)}$ the system noise is approximated based on the is the sampling interval, the spectral density matrix and standard deviations of the driving noise of the system.

In the case of integrating code positioning with SADDCP, the vector of measurements includes six observations; three observations from the absolute code positioning of epoch k ($X_{(k)}$, $Y_{(k)}$, $Z_{(k)}$) and three values describing the 3D changes in

positioning between epochs k & $k+1$ ($dX_{(k, k+1)}$, $dY_{(k, k+1)}$, $dZ_{(k, k+1)}$). As there are no unknowns, the $Z_{(k)}$ and $U_{(k)}$ are the same vector, H is a (6*6) unit matrix and $v_{(k)}$ equals zero.

$$Z_{(k)} = U_{(k)} = [X_{(k)} \ Y_{(k)} \ Z_{(k)} \ dX_{(k, k+1)} \ dY_{(k, k+1)} \ dZ_{(k, k+1)}]^T$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As for the dynamic model, the system state vector at the epoch $(k+1)$ includes three observations from the absolute code positioning for this epoch ($X_{(k+1)}$, $Y_{(k+1)}$, $Z_{(k+1)}$) and three values describing the 3D changes in positioning between this epoch and the previous one ($dX_{(k, k+1)}$, $dY_{(k, k+1)}$, $dZ_{(k, k+1)}$). The system state vector can be defined as:

$$U_{(k+1)} = [X_{(k+1)} \ Y_{(k+1)} \ Z_{(k+1)} \ dX_{(k, k+1)} \ dY_{(k, k+1)} \ dZ_{(k, k+1)}]^T$$

The smoothed code positioning in epoch $k+1$ can be defined as:

$$\begin{aligned} X_{(k+1)} &= X_{(k)} + dX_{(k, k+1)} \\ Y_{(k+1)} &= Y_{(k)} + dY_{(k, k+1)} \\ Z_{(k+1)} &= Z_{(k)} + dZ_{(k, k+1)} \end{aligned}$$

To relate the state vectors in epoch $k+1$ with that in epoch k , the state transition matrix (M) should be written as:

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The standard deviations for the carrier phase measurements used in the system noise vector $W(k)$ in the dynamic model is computed using the signal to noise ratio obtained directly from the receiver RXMRAW message. The standard deviation of carrier phase measurements tends to round about millimeters in multipath-free environment and reaches the quarter of the GPS wavelength (5cm) as a maximum value [4]. This is not the case with stand-alone code measurements where the precision level fluctuates between a few and tens of meters. In this paper, the differences between the computed smoothed position at epoch $(k+1)$ and the code position at the same epoch will be used as an indication on the quality of the code measurements at the next

epoch. The integration of SADDCP and stand-alone code positioning is presented in the following workflow diagram which has been implemented by the author in Matlab.

2.3 Cycle Slip Detection

As the integration solution based on carrier phase measurements, cycle slips can happen when losing the connection between satellite and receiver. In this case, a random integer number of cycles is added to the carrier measurements in the GPS data file. Cycle slips can be detected by comparing the differences between the code and carrier phase measurements of two adjacent epochs which is the method used in SADDCP Matlab code. Code positioning provides continuous solution as long as four satellites can

be detected and it is not necessary for the same satellites to be detected adjacent epoch where the code position in each moment is independent solution. When adequate number of satellites is available, the carrier measurement including cycle slip is removed for more precise solution [1]. As the receiver clock error is removed with SADDCP, the minimum number of satellite required to get the relative positioning is 3 [4]. If the number of free-cycle slips phase measurements is less than 3, pseudo-range measurements are used to fix the gap in phase measurements to be used in the relative solution. Satellite residuals should be statically investigated to remove the outliers. Data Snooping Method is used with SADDCP. Firstly, the covariance matrix of satellite residuals should be determined. Then, the square roots of the diagonal elements are extracted giving the standard deviation of each observation. The residual of each observation is divided with its standard deviation. This value should fluctuate from 0 to 3 depending on the required confidence level. With 99% confidence level chosen in this paper, the critical value is nearly 2.6 so any value bigger than this is detected as an outlier with 1% probability of rejection the observation when it should be accepted (type 1 error) [8].

2.4 The Integration Solution Reference Point

As the integration solution works on providing absolute positioning, at least one absolute 3D georeferenced point should be available based on code positioning. This reference point can be at any place throughout the trajectory and should be

as accurate as possible where the quality of the integration solution depends mainly on this point. Providing more than one accurate point through the trajectory can help on reducing the cumulating of SADDCP relative positioning where each position is based on the previous one. In general, two types of absolute stand-alone code positioning can be determined, namely: static and mobile. In static solution, the whole code observations are solved to get one solution using least squares or Kalman filtering, whereas mobile solution provides one solution for each epoch. Static solution is more precise than mobile solution where where the more observations, the better outputs. This is because increasing the number of observations for measuring the same variables tends to close the final solution to the absolute mean value which is achieved using infinity number of observations. This absolute mean value is equal to the true value when removing the systematic errors. Increasing the number of observations in the case of static solution helps to remove the less precise measurements with residuals bigger than 1σ . Although this can increase the solution quality significantly, the confidence level decreases the and as a consequence, the probability of rejection the observation when it should be accepted increases. These two types of reference points will be tested for the integration solution besides using known reference point to investigate the advantages and limitations of each one.

2.5 The Integration Solution Workflow

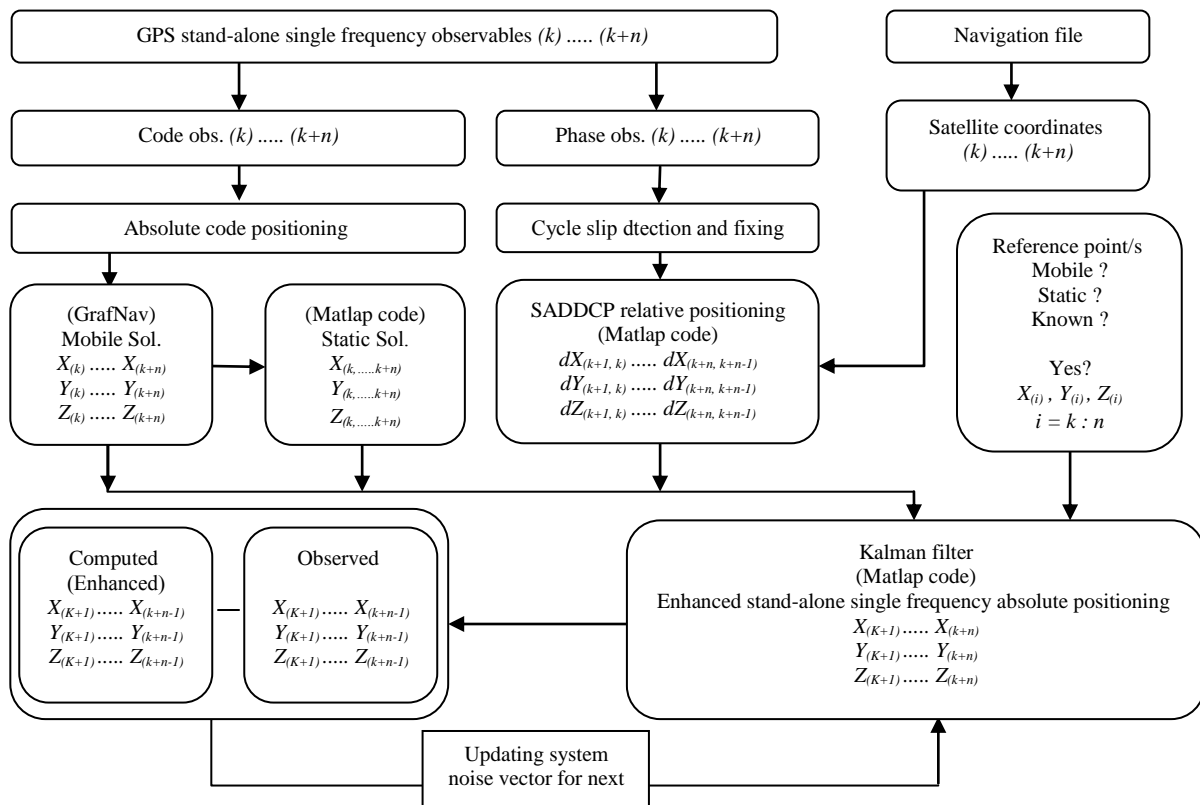


Fig. (1): Workflow diagram of SADDCP/code positioning integration

3. TEST AND RESULTS

The integration of SADDCP/code positioning has been tested in different GPS environments. Leica dual frequency GNSS receiver has been used with the reference base station of Benghazi University, Libya. The test has been divided into two parts; the first is static for 30 minutes and the second is kinematic for another 30 minutes. The kinematic part includes passing throughout areas with limited coverage GPS and surrounded by high buildings to investigate the effect of cycle slips and high multipath on the quality of integrated positioning. Figures 1 shows parts of the test site. GrafNav software has been used for processing the collected GPS raw data to provide mobile single point positioning (stand-alone code positioning) as well as dual frequency carrier phase DGPS solution, which has been used as a reference for evaluating the integration solution.

For the static part, the navigation solution for each epoch has been compared to the static carrier phase DGPS solution (one solution for all epochs). The integrated navigation solution has been tested

based on mobile, static and known reference point as described in 2.3 & 2.4. For the kinematic part, the navigation solution for each epoch has been compared to the kinematic carrier phase DGPS solution (one solution for each epoch). The integrated navigation solution has been tested based on mobile, static and known reference point as in static part. Table 1 and Table 2 illustrate the accuracy and precision of the integration solutions as well as the individual stand-alone code positioning and SADDCP relative positioning in static and kinematic cases, respectively. It should be mentioned that for relative positioning, just the precision level can be investigated, which can be used as an indication on the solution accuracy. The precision of individual SADDCP solutions has been determined based on the separate solution for each epoch in static case and using cumulative positioning for kinematic case. Figure 2 and Figure 3 show the 3D errors of the integration solutions and the stand-alone code solution for static and kinematic cases, in this order.

Table (1): Accuracy and precision of integration solutions, stand-alone code solution and SADDCP relative solution (Static case)

Integration solution Mobile reference point		Integration solution Static 1σ fixed reference point		Integration solution Known reference point		Mobile code solution		SADDCP Individual
3D Accuracy (m)	3D Precision (m) 3σ	3D Accuracy (m)	3D Precision (m) 3σ	3D Accuracy (m)	3D Precision (m) 3σ	3D Accuracy (m)	3D Precision (m) 3σ	3D Precision (m) 3σ
5.172	0.525	2.086	0.525	0.832	0.525	6.382	11.864	0.012
Accuracy of static code solution based on all obs.		Accuracy of static code solution based on 3σ obs.		Accuracy of static code solution based on 2σ obs.		Accuracy of static code solution based on 1σ obs.		
6.382 m		5.209 m		3.713 m		2.247 m		

Table (2): Accuracy and precision of integration solutions, stand-alone code solution and SADDCP relative solution (Kinematic case)

Integration solution Mobile reference point	Integration solution Static 1σ fixed reference point	Integration solution Known reference point	Mobile code solution	SADDCP cumulative
3D RMSE (m)	3D RMSE (m)	3D RMSE (m)	3D RMSE (m)	3D RMSE (m)
9.203	4.346	2.081	15.022	2.924

4. DISCUSSION

It is clear from table 1 that the precision of SDDCP residuals is within 1 cm for the relative positioning between each two epochs. These residuals can be attributed to the errors that reduced not removed after applying the double differences. Examples for such errors are satellite clock error which is affected by the stability of the satellite clock between two epochs, ionospheric and tropospheric delays which might change across the interval. Multipath is an expected source of errors in SADDCP where reducing this type of errors depends just on the correlation over time which is affected by the change in satellite constellation over time. Receiver measurements noise is increased with the double difference, but such error tends to round about parts of millimeters [4]. Number of cycle slip has been detected based on the filter discussed in 2.3 and as the static test has been applied in open sky with significant number of satellites, the observations including cycle slips have been removed with 99% confidence level. In the case of cumulating SADDCP for getting continuous relative solution starting from known point, table 2 and figure 2 show that the errors in SADDCP have cumulated with time reaching nearly 8 m by the end of trajectory with root mean square error of about 3 m. cumulating the errors in the adjacent epochs

relative positioning is the main limitation which reflects the need for this technique to be integrated with code positioning as the solution is independent for each epoch.

As for the code positioning in the static of the test, the mobile solution (one solution for each epoch), as shown in table 1 and figure 2, has been imprecise with 3D vector accuracy of several meters comparing to the static dual frequency DGPS solution (one solution). This is expected due to the typical pseudo-range source of errors mentioned before. When moving in the kinematic part, the quality of the code positioning has degraded more and more as seen from figure 3 and table 2. This can be attributed mainly to the high multipath effect and the poor satellite geometry in some parts of the trajectory where the area is surrounded by high buildings.

The site is a suitable environment for multipath and reflecting signals where it is surrounded by buildings. Multipath effect can reach several meters in high multipath environment using code measurements and several centimeters for carrier phase. The system used in the test is provided with right hand circular polarization pinwheel antenna to mitigate the effect of the reflected signals. The transmitted signals from satellites are right hand circular polarization and this polarization is changed based on the reflection angle and the number of reflections. Therefore, using an antenna

with right hand circular polarization helps to reject the reflected signals which tend to have right hand circular polarization. However, the left hand circular polarization reflected signals, which are reflected twice or more are received by the GPS antenna and the receiver may not be able to deal with such signals. In some receivers, such as that used in the test, narrow correlation technique is used to deal with the received multipath signals where the direct signal can be signified from that reflected based on the arriving time and the signal strength. In the case of just receiving the reflected signal where the satellite is hidden by abstracts, it tends to be difficult to detect the multipath effect even using such technique [9] [4].

Furthermore, it has been noticed that in this site, the number of satellite cycle slips has increased comparing to the other parts of trajectory. Beside the effect of high buildings on masking the satellites from time to time causing cycle slips, the high multipath can also disturb the correlation between the codes of direct signals and those generated by the receiver and as a consequence, connection might be lost resulting cycle slips. The multi-reflected signals might have right hand circular polarization which can be received by the antenna without any rejection. If the direct signal is already received with the reflected signal, the code correlation between satellite and receiver could be affected leading the connection to be lost. If just the multipath signal is received without the direct one, two scenarios are expected; in the first, just one reflected signal is received and the connection with the satellites is not affected and the second scenario is when more than one reflected signal from the same satellite are received by the antenna. In this case, the connection with the satellite might be affected depending on the strength of the received signals. Moreover, increasing the level of multipath in the area often creates an electromagnetic noise around the antenna which can affect the antenna directivity. This means that the radiation pattern will not be the same in all directions and accordingly signals facing the low gain antenna side may not be received degrading the positioning quality [4].

Some post processing software, such as GrafNav utilizes the satellite residuals to mitigate the multipath effect where satellites with significant residuals are removed from the calculations and the position is recalculated again. However, this can be applied with static carrier

phase DGPS using dual frequency receivers and short baseline where the majority of errors are cancelled out or mitigated to great extent, except that of multipath and receiver noise. In the case of stand-alone code positioning, it is hard to use satellite residuals for detecting the multipath effect where the ionosphere effect might have an effect on the satellite residuals more than that of multipath. However, removing observations with residuals bigger than three times the standard deviation of the whole observations always can improve the results [8].

The second reason for degrading the results in urban areas with high buildings is the satellite geometry, where just high elevation satellites can be detected making the solution geometry very weak. This can increase the altitude errors considerably due to the small intersection angles between satellites. The results show also that the plan quality for all solutions has been better than the altitude quality which can be attributed to the satellite geometry. Theoretically, the best overall quality can be achieved with 5 satellites; four are distributed with 90 degrees in azimuth and at 40 to 50 elevation angle and one is overhead. Increasing this last helps to achieve better plan quality and leads the vertical quality to be reduced and vice versa. On the other hand, using low elevation satellites tends to be avoided affecting the attitude quality. This is because GPS signal path of the low elevation satellite passes through more atmosphere than the vertical satellite. This is important where the positioning calculation in GPS is based on the assumption that GPS signal travels in a vacuum. Therefore, signals of low elevation satellites have more delay and consequently give less precise results. Also, passing the signal through the atmosphere for longer distances tends to make it noisier and not clean affecting the goodness of data. Results show also that the quality in E-W direction, generally, better than N-S direction which can also attributed to the satellite geometry. In GPS, the number of satellites in E-W direction is more than that of N-S direction due to the inclination angle of the satellite orbits. GPS satellite orbits have 55 inclination angles which mean that the satellites fluctuate in the area between +55 degree and -55 degree from the Equator. This means that in areas located above this degree, the majority of satellites are locate overhead and on E, W and S directions and a few satellites can be detected in the north with low elevation angles due to the height of the satellite above the Earth [1][4][9].

Table 1 illustrates also the accuracy of static code solution based on different levels of σ filtration. The results show that the more restrictions, the more precise results and as a consequence, the more accurate solution. When using all observations, gross errors, caused for example by high multipath and ionosphere, are considered in the solution degrading the results considerably. With (3σ , 2σ , and 1σ), (99.7%, 95%, and 68.2%) of the observations are used, respectively. The static code solution based on 68.2% of the observations has been used as one of the reference start point with the integration solution. With precise measurements and neglected systematic errors, even small number of observations can provide results close to the absolute mean value and increasing the number of such precise observations makes increases the level of reliability and may not increase the accuracy.

The results show the quality of the integrated solutions based on three started reference points. Using different points with the integration solution can affect only the accuracy level where the more accurate reference point, the closer the integration solution to the true solution. This means that, the reference point works as a displacement transformation factor. In both static and kinematic parts, the integrated solution based on known point has had the best results, solution based on static referenced point came second and that based on mobile solution has been the last. The results show the ability of the integration solution to exploit the precision of the double differencing relative solution, the high capability of carrier phase on reducing the multipath effect, and the absolute and independently of code solution. However, as it is difficult to get a known point to start with, the best option is to use static based reference point/s where the receiver should be fixed at one or more points throughout the trajectory for a period based on the required results quality level.

5. CONCLUSION

In this paper, SADDCP has been used to enhance the performance of stand-alone GPS code positioning, where the two positioning techniques have been integrated using Kalman filter. The precise relative positioning provided by SADDCP have been utilized to smooth the absolute low accurate stand-alone GPS code positioning, providing enhanced absolute single frequency stand-alone GPS positioning. The integration

solution has been tested in different GPS environments for reliable investigations. The results show ability of the integration solution on gathering the advantages of the two positioning techniques providing enhanced absolute single frequency stand-alone positioning. The integration solution utilizes the high precision of the double differencing relative solution and high capability of carrier phase measurements on reducing the multipath effect to enhance the degraded imprecise code solution. On the other hand, the absolute, independently and continuity of code solution to overcome the limitation of SADDCP in terms of being relative, dependent, error cumulative, and susceptible to cycle slips. The main limitation of the integration system is the need for at least one accurate reference point at any place throughout the trajectory which might be obtained by static code solution for a period related to the required solution quality level.

REFERENCES

- 1- Hofmann-Wellenhof, B., Lichtenegger, H. and Wasle, E., 2007. GNSS global navigation satellite systems: GPS, GLONASS, Galileo, and More. Springer Science & Business Media.
- 2- Li, Y., Zhang, K., Roberts, C. and Murata, M., 2004. On-the-fly GPS-based attitude determination using single- and double-differenced carrier phase measurements. *GPS Solutions*, 8(2), pp. 93-102.
- 3- Amami, M.M. and Smith, M.J. and Kokkas, N., 2014. Low cost vision based personal mobile mapping system. *ISPRS- International Archives of The Photogrammetry, Remote Sensing and Spatial Information Sciences*, XL-3/W1 .pp. 1-6. ISSN 2194-9034
- 4- Amami, M., 2015. Low cost vision based personal mobile mapping system. Doctoral dissertation, University of Nottingham, UK.
- 5- Blake, S., 2007. Heave compensation using time-differenced carrier observations from low-cost GPS receivers. Doctoral dissertation, University of Nottingham, UK.
- 6- Hoffman, B., Wellenhof, H. L. and Collins, J., 2001. *GPS theory and practice*. Springer
- 7- Groves, P. D., 2013. *Principles of GNSS, inertial and multi-sensor integrated navigation systems*. Artech House.
- 8- Cross, P. A., 1990. Advanced least squares applied to position-fixing, working papers. Polytechnic of East London, Department of Land Surveying
- 9- Grewal, M. S., Andrews, A. P. and Bartone, C. G., 2013. *Global navigation satellite systems, inertial navigation, and integration*. John Wiley & Sons