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NUMERICAL ANALYSIS OF GLUED LAMINATED TIMBER WITH UNEQUAL LENGTHS COMPONENTS

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ABSTRACT

An innovative study has been carried out on timber beams strengthened mechanically by two external layers attached to their tension and compression sides with glue. This study is based on the individual behavior of each component of the laminate section. An approach has been developed to simulate the behavior of such beams. The equations are formulated and solved numerically using finite difference method and computational analysis.

The interaction efficiency indicated by slip and deflection calculations between the three layers in a timber beam has been considered thoroughly, from which the effect of some parameters such as layer length upon the behavior of such beams are studied.

New equations are proposed for such system to calculate the deflection of laminated timber beams.

KEYWORDS: Laminated Timber; interaction between wooden layers; Finite Deference; Epoxy.

1. INTRODUCTION

Glued laminated limber members are produced in laminating plants by gluing together dry lumber, normally of 2-in. or 1-in. nominal thickness, under controlled conditions of temperature and pressure. Glued laminated timber members are typically produced in three appearance classifications. Premium, Architectural, and Industrial, though Industrial Special is also available[1]. Premium and Architectural beams are higher appearance classifications and are surfaced for a smooth, beautiful finish. Industrial appearance beams are normally used in concealed applications or in construction where appearance is not important. Industrial Special appearance beams are typically used for headers. Design values for the glued laminated timber member are independent of the appearance classifications.

In 1968, Goodman [2] produced one of the earliest theories to analyze the behavior of layered beam systems with interlayer slip. In this theory, a three -layered system is used; all layers are considered to have the same mechanical properties throughout. The connectors for the beam are assumed to be equally space and of equal strength. The governing differential equation is of the second order in terms of axial force. Nine experiments with layered wood beams are performed to verify his theory. In 1986, McCutcheon [3] resented a simple procedure for computing the composite stiffness of wood bending member with sheathing attached no rigidly to one or both edges .and he modify the axial stiffness's of the flanges and then compute the stiffness of the resulting T-beam or I-beam by the transformed area procedure an accounting for interlayer slip , so his test data agreed very closely with theoretical predictions .also the framing members will be assumed to be the principal load-carrying elements in the resulting T-beams and I-beams , the method will assume that all materials , including connectors behave linearly and that the interlayer stiffness is much lower than the stiffness's of framing members (web) or sheathings (flanges). Xu et.al. [4] in 2012 studied the behaviour of glued-in ribbed steel bars in timber beam-to-beam connections experimentally and numerically. The experimental results exhibit two different failure modes due to the internal stiffness distribution between steel and timber in bending. They are used to validate a 3D finite element model developed considering the actual geometry of the connections. Their model considers the elasto-plastic behaviour of steel and the orthotropic elasto-plastic behaviour of timber.

In 2015 Umaima and Arya [5] presented a study to determine analytically the flexural properties of

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glued laminated timber beam (glulam) with different thickness of lamina and jointed lamina; it was compared to solid beam. Their research used Rubber wood (Hevea brasiliensis). Glulam beam was divided into three groups based on the thickness of lamina, 20 mm, 15 mm and 10 mm respectively. Solid beam was also modelled besides glulam beam. Jointed wood with finger outside maximum moment zone in the bottom lamina was also modelled. In 2013 Fink et.al. [6] modelled the probabilistic representation of the material properties of Glued Laminated Timber (GLT) that considers the natural growth characteristic of timber. Further, 24 GLT beams with well-known local material properties are produced and tested in order to validate the model.

2. ASSUMPTIONS

A single theory of interaction taking both slip and uplift effects into account is presented assuming bending theory but ignoring shear lag effects. The formulations of existing works take account of either differential strain only, or of differential deflections only, but not both together, in this work of glued laminated timber beam indicate that both slip and uplift occur simultaneously where the elastic connection is flexible. In addition, it is assumed that the rate of change of the axial force is directly proportional to slip, and uplift force is directly proportional to differential deflection. This last assumption implies the existing of two Modula, one that depends on the ability of the connectors to resist slip (Ks), and the other, which depends on the resistance of the connectors to uplift in parts where separation occurs (Kn),

3. FORMULATION

Assuming an element of length (δx) of timber section shown in Figure (1) and satisfying the equilibrium of horizontal and vertical forces for timber elements , the following will be obtained:

$\sum Fx = 0,$ then,

Longitudinal equilibrium of upper timber layer (layer) gives :-

$$N_{a,x} = -q_1 \quad \text{.........(1)}$$

Similarly for timber core layer and lower timber layer, respectively

$$N_{a,x} - N_{c,x} = q_2 - q_1 \quad \text{..........(2)}$$

$$N_{c,x} = -q_2 \quad \text{...........(3)}$$

$\sum Fy = 0$,then,

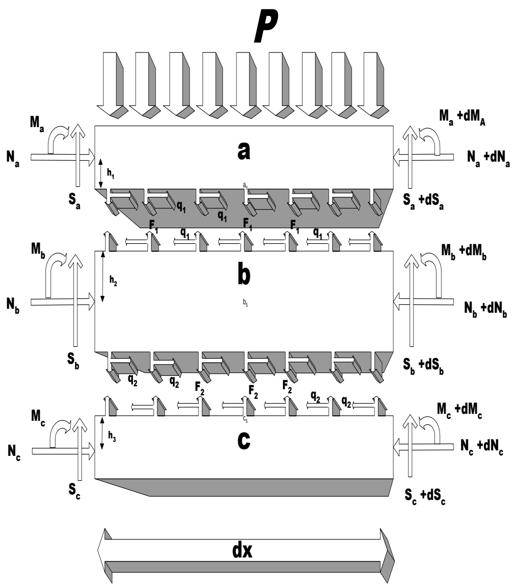

Fig.(1): composite finite element

$$S_{a,x} = P + F_1 \text{ .... (4) For upper layer}$$

$$S_{b,x} = F_1 - F_2 \text{ . ... (5) For middle layer}$$

$S_{c,x} = -F_2$ ……………… (6) For lower layer

Considering moment equilibrium of the timber core layer and two timber layers about points, a_5, b_5 and c_5 which represent the center of the three layers respectively, gives:-

$$M_{a,x} = -S_a - N_{a,x}.h_1 \quad \text{………………… (7)}$$

$$M_{b,x} = -S_b - N_{a,x}.d_1 - N_{c,x}.d_2 \quad \text{……….. (8)}$$

$$M_{c,x} = -S_c - N_{c,x}.h_3 \quad \text{……………..….. (9)}$$

In which subscripts (a), (b) and (c) denoted the upper middle and lower layer respectively subscripts (1) and (2) denote the interface between upper timber layer and timber core layer and the interface between the timber core layer and the lower timber layer respectively. (V) Denotes the vertical shear at a section, distance (x) from the support. Hence

$$V = S_a + S_b + S_c \quad \text{……………….. (10)}$$

Furthermore, the external moment at any cross section of the strengthened beam will be resisted by the sum of the moments in (a),(b),(c) layers plus the triple arising from the axial forces in the three element.

$$M = Ma + Mb + Mc + N(d_1 + d_2) \quad \text{… (11)}$$

$$d_1 = h_1 + h_2 \qquad d_2 = h_2 + h_3$$

N = Na = Nc

Where d_1,d_2 in the distance between the centurions of the upper-middle- and lower layer respectively, differentiating equation. (7),(8) and (9) once with respect to x then replacing values of $S_{a,x}$, $S_{b,x}$ and $S_{c,x}$ from equation (4) , (5) and (6) so, these equations became:-

$$M_{a,xx} = -(P + F_1) - N_{a,xx}.h_1 \quad \text{…………..….(12)}$$

$$M_{b,xx} = -(F_1 - F_2) - N_{a,xx}.d_1 - N_{c,xx}.d_2 \quad \text{………..(13)}$$

$$M_{c,xx} = -F_2 - N_{c,xx}.h_3 \quad \text{…....…..……(14)}$$

and in compatibility equations the curvature of the reinforced timber core layer and two timber layers is ($W_{b,xx}$) and ($W_{a,xx}$,$W_{c,xx}$) respectively, can be defined from elastic beam theory as below:

$$W_{a,xx} = \frac{M_a}{\beta_1} \quad \text{…………..(15)}$$

$$W_{b,xx} = \frac{M_b}{\beta_2} \quad \text{…………..(16)}$$

$$W_{c,xx} = \frac{M_c}{B_3} \quad \text{…………..(17)}$$

Where:-

$$\beta_1 = E_{up}.I_{up}$$

$$\beta_2 = E_{co}.I_{co}$$

$$\beta_3 = E_{lp}.I_{lp}$$

Therefore relating equations (12),(13),(14),(15),(16) and (17) will give

$$\frac{d^4 w_a}{dx^4}\beta_1 = -\frac{d^2 N_a}{dx^2}h_1 - (P + F_1) \quad \text{…………..…. (18)}$$

$$\frac{d^4 w_b}{dx^4}\beta_2 = \left(\frac{d^2 N_a}{dx^2}.d_1 + \frac{d_2 N_c}{dx^2}.d_2\right) - (F1 - F2) \quad \text{…………. (19)}$$

$$\frac{d^4 w_c}{dx^4}\beta_3 = -h_3\frac{d_2 N_c{}^*}{dx^2}.h_3 - F_2 \quad \text{……… (20)}$$

The tension peeling forces (F), arises from the deformation of the elastic connection due to differential displacement between the three layers at their two interfaces, so that

$$F_1 = Kn(Wa - Wb) \quad \text{……………………..….(21)}$$

$$F_2 = Kn(Wb - Wc) \quad \text{……………………..….(22)}$$

Where (Kn) is the normal stiffness per unit length for vertical displacement. In addition, we can differentiate equations (21) and (22) four times with respect to x and after that, we substitute (d^4Wa/dx^4), (d^4Wb/dx^4), (d^4Wc/dx^4) from equations (18), (19) and (20) yield:-

$$\frac{d^4F_1}{dx^4}+Kn\left(\frac{1}{B_1}\right).P-Kn\left[\frac{d_1}{B_2}-\frac{h_1}{B_1}\right]\frac{d^2N_a}{dx^2}+Kn\left[\frac{d_2}{B_2}\right]\frac{d^2Nc}{dx^2}+F_1\left[\frac{1}{B_1}+\frac{1}{B_2}\right]Kn-F_2\left[\frac{1}{B_2}\right]Kn=0 \quad \ldots\ldots(23)$$

$$\frac{d^4F_2}{dx^4}+Kn\left[\frac{d_1}{B_2}\right]\frac{d_2Na}{dx^2}-Kn\left[\frac{h_3}{B_3}-\frac{d_2}{B_2}\right]\frac{d^2Nc}{dx^2}+F_1\left[\frac{1}{B_2}\right]Kn+F_2\left[\frac{1}{B_3}-\frac{1}{B_2}\right]Kn=0 \quad \ldots\ldots(24)$$

Differentiating equations (21) (22) twice with respect to x and substituting for curvature in terms of moment and flexural rigidity and rearranging give,

$$\frac{1}{Kn}.\frac{d^2F_1}{dx_2}=\frac{M_a}{B_1}-\frac{M_b}{B_2} \quad \ldots\ldots(25)$$

$$\frac{1}{Kn}.\frac{d^2F_2}{dx^2}=\frac{M_b}{B_2}-\frac{M_c}{B_3} \quad \ldots\ldots(26)$$

from equation (11) and equation (25) and (26) the moment in the three layers will be :-

$$\frac{Ma}{B_1}=\left[\frac{M-N(d_1+d_2)}{B_2}+\frac{F1,xx}{Kn}\right].\frac{B_2}{B_1+B_2} \quad \ldots\ldots (27)$$

Mb due to upper intrface

$$\frac{Mb}{B_2}=\left[\frac{M-N(d_1+d_2)}{B_1}+\frac{F_1,xx}{Kn}\right].\frac{B1}{B_1+B_2} \quad \ldots\ldots (28)$$

Mb Due to lower interface

$$\frac{Mb}{B_2}=\left[\frac{M-N(d_1+d_2)}{B_3}+\frac{F_2,xx}{Kn}\right].\frac{B_3}{B_2+B_3} \quad \ldots\ldots (29)$$

$$\frac{Mc}{B_3}=\left[\frac{M-N(d_1+d_2)}{B_2}+\frac{F_2,xx}{Kn}\right].\frac{B_2}{B_2+B_3} \quad \ldots\ldots (30)$$

the rate of change of slip at the first interface at any point equal to the differential strain at this point , Hence :-

$$U_{ab},_x=\varepsilon_a-\varepsilon_b \quad \ldots\ldots (31)$$

and we can define (ε_a) and (ε_b) as

$$\varepsilon_a=W_a,xx.h_1-\frac{N}{\alpha_1} \quad \ldots\ldots (32)$$

$$\varepsilon_b=W_b,xx.h_2-\frac{N}{\alpha_2} \quad \ldots\ldots (33)$$

$$\alpha_1=E_{up}.A_{up}$$

$$\alpha_2=E_{co}.A_{co}$$

Similarly, for the second interface the slip is -

$$U_{bc},x=\varepsilon_b-\varepsilon_c \quad \ldots\ldots (34)$$

$$\varepsilon_b=W_b,xx.h_2-\frac{N}{\alpha_2} \quad \ldots\ldots (35)$$

$$\varepsilon_c=Wc,xx.h_3-\frac{N}{\alpha_3} \quad \ldots\ldots (36)$$

$$\alpha_3=E_{lp}.A_{lp}$$

Where (h_1) and (h_2), (h_3) are the distance between the interface and the centered of the upper timber layer and timber core layer and the lower timber layer respectively.
So we can write equations (31) and (34) as:

$$U_{ab},x=\frac{M_a.h_1}{B_1}+\frac{M_b.h_2}{B_2}-\frac{N}{\alpha_1}-\frac{N}{\alpha_2} \quad \ldots\ldots (37)$$

$$U_{bc},x=\frac{M_b.h_2}{B_2}+\frac{M_c.h_3}{B_3}-\frac{N}{\alpha_2}-\frac{N}{\alpha_3} \quad \ldots\ldots (38)$$

Substituting for the values of curvature of upper and lower elements from equations (27), (28), (29), and (30) respectively then equations (37) and (38) becomes

$$U_{ab},x=-N\left[\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{d_1^2+d_1d_2}{B_1B_2}\right]-\frac{1}{Kn}\left[\frac{B_2h_1-B_1h_2}{B_1+B_2}\right]F_{1,}xx+M\left[\frac{d_1}{B_1+B_2}\right] \quad \ldots\ldots (39)$$

$$U_{bc},x\approx M\left[\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\frac{d_2^2+d_1d_2}{B_2+B_3}\right]-\frac{1}{Kn}\left[\frac{B_3h_2-B_2h_3}{B_2+B_3}\right]F_2,xx+M\left[\frac{d_2}{B_2+B_3}\right] \quad \ldots\ldots (40)$$

Hence, shear flow (q) can be related to the interface slip between the two elements, therefore,

$$q_1=q_2=\frac{U_{cs}.K}{s} \quad \ldots\ldots (41)$$

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Where (K) is the shear stiffness of shear connectors and (s) in the spacing between them.

Then relating equations (41), (1) and (3)

$$U_{ab}, x = \frac{1}{k_s} \cdot \left[\frac{d_2 Na}{dx_2} \right] \quad \text{……………………..} \quad (42)$$

$$U_{ac}, x = \frac{1}{k_s} \cdot \left[\frac{d_2 Nc}{dx_2} \right] \quad \text{…………………………..} \quad (43)$$

Equating equations (42) and (43) with (39) and (40) yields

$$-\frac{d^2 Na}{dx^2} - N\left[\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{d_1^{\,2} + d_1 d_2}{B_1 + B_2} \right].Ks - \frac{Ks}{Kn}\left[\frac{B_2 h_1 - B_1 h_2}{B_1 + B_2} \right] \frac{d^2 F_1}{dx^2} + Ks.\frac{d_1}{B_1 + B_2}.M = 0 \quad \text{…………………………..} \quad (44)$$

4. NUMERICAL SOLUTION

We should bear in mind that all methods of structural analysis are essentially concerned with solving the basic differential equation of equilibrium and compatibility, although in some of the methods this fact may be obscured. Analytical solutions are limited to the cases when the load distribution, section properties and boundary conditions can be described by mathematical expressions, but for complex structure like our care, numerical methods are in general a more practical means of analysis, so we solve equations (23), (24), (44) and (45) numerically using finite difference methods of various derivatives. This method will save time and effort as a personal computer can be used to apply the final solution to difference loading can dittoing. In order to achieve higher accuracy in the solution of the differential equation by finite differences the four equation contain derivatives of fourth and second order which can be expressed in a form called central differences form, and solved by a method suggested by Fox for solving two-point boundary value problems involving differential equation of orders higher than two. In order to achieve higher accuracy in the solution of the differential equations by finite differences, the equations are rearranged such that no derivative higher than second order occurs. Thus assuming an intermediate function modifies the equations (46) and (47):

$$U_1 = \frac{d^2 F_1}{dx^2} \quad \text{……………………………………………..} \quad (46)$$

$$U_2 = \frac{d^2 F_2}{dx^2} \quad \text{………………………………………..…….} \quad (47)$$

So that the six differential equations, (23,24), (44,45) and (46,47), with five unknown variables, (indeed four unknown for each interface) (F_1),(F_2) (N), and (U_1),(U_2) can be written in the following forms,

$$U_{1,xx} - \gamma_0.F_1 + \gamma_1.F_2 - \gamma_2.N_{,xx} = -\gamma_3.p \ \text{…} \ (48)$$

$$U_{2,xx} + \gamma_1.F_1 - \gamma_4.F_2 - \gamma_5.N_{,xx} = 0 \quad \text{……………} \ (49)$$

$$-N_{,xx} - \gamma_6.N - \gamma_7.U_1 = -\gamma_8.M_b \ \text{……..} \ (50)$$

$$-N_{,xx} - \gamma_9.N - \gamma_{10}.U_2 = -\gamma_{11}.M_b \ \text{………..} \ (51)$$

$$F_{1,xx} - U_1 = 0 \ \text{…………………………………} \ (52)$$

$$F_{2,xx} - U_2 = 0 \ \text{…………………………} \ (53)$$

where:

$$\gamma_0 = \left[\frac{1}{B_1} - \frac{1}{B_2} \right] Kn$$

$$\gamma_1 = \left[\frac{1}{B_2} \right] Kn$$

$$\gamma_2 = Kn\left[\frac{d_1}{B_2} - \frac{h_1}{B_1} - \frac{d_2}{B_2} \right]$$

$$\gamma_3 = Kn\left[\frac{1}{B_1} \right]$$

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$$\gamma_4 = Kn\left[\frac{1}{B_3}+\frac{1}{B_2}\right]$$

$$\gamma_5 = Kn\left[\frac{h_3}{B_3}-\frac{d_2}{B_2}+\frac{d_1}{B_2}\right]$$

$$\gamma_6 = \left[\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{d_1^2+d_{1.}d_2}{B_1+B_2}\right].Ks$$

$$\gamma_7 = \left[\frac{B_{2.}h_1-B_1.h_2}{B_1+B_2}\right].\frac{Ks}{Kn}$$

$$\gamma_8 = \left[\frac{d_1}{B_1+B_2}\right].Ks$$

$$\gamma_9 = \left[\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\frac{d_2^2+d_{1.}d_2}{B_3+B_2}\right].Ks$$

$$\gamma_{10} = \left[\frac{B_{3.}h_2-B_2.h_3}{B_3+B_2}\right].\frac{Ks}{Kn}$$

$$\gamma_{11} = \left[\frac{d_2}{B_3+B_2}\right].Ks$$

equation (47) , (48) contain derivatives of second order in term of N,F_1,F_2,U_1 and U_2 which can be expressed in finite central difference form, using three node points as given below

$$Y_{i,xx} = \frac{Y_{i-1}-2.Y_i+Y_{i+1}}{\lambda_2} \text{ (Central) } \ldots\ldots\ldots (54)$$

In which, λ is the node division, y in the dependent variable,

(i) Number of node. In addition, substituting the above formula finite difference into equations (48) and (53) respectively.

5. THE BOUNDARY CONDITIONS

The finite difference formulation requires introducing external node on each end of the timber layer; because, the final differential equations are of second order, see Fig. (4.2). Therefore, eight boundary conditions are required to be established at the beam (four boundary conditions for each interface) .For the case of a simply supported beam, the lengths of the three elements are unequal; therefore we use the following boundary conditions,

1. From equations (25),(26), at (x = a+b)and (x=a), (Ma & Mc = 0), free end of upper and lower timber layers, the first and second boundary conditions yield the following,

$F_{1,xx} = \gamma_1 .M_b$ ………………………………..(55)

$F_{2,xx} = \gamma_1$ ……………………………………(56)

2. By differentiating equations (21),(22) thrice with respect to (x), and substituting for the values of(Ma,Mb & Mc) from equations (7) ,(8) and (9) respectively, that will lead to a new two boundary conditions at (x =a)and (x=a+b),which are,

$$U_{1,x} + \gamma_1.(h_1 - h_3).N = -\gamma_1.S_b \ldots\ldots\ldots (57)$$

$$U_{2,x} + \gamma_1.(h_1 - h_3).N = -\gamma_1.S_b \ldots\ldots\ldots\ldots (58)$$

3. From equations (27), (30) the strain in timber layers tends to zero at (x=a+b) and (x=a) then, the fifth and sixth boundary conditions are:

$$Na_{,xx} - \frac{Ks}{\alpha_2}\times N = \frac{Ks}{B_2}\times M_b \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

$$Nc,_{xx} - Ks/\alpha_2 \times N = Ks/\beta_2 \times Mb \ldots\ldots\ldots\ldots (60)$$

4. We can obtain the seventh and eighth boundary conditions by taking the fifth derivatives of (F_1 and F_2) as a (mathematical trick) at each interface with respect to x yield:

$$F_{1,xxxxx} = U_{1,xxx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

$$F_{2,xxxxx} = U_{2,xxx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Similar boundary conditions will be used in the far end of the timber layer (i.e. at x = (l_{up}+a+b) and (x=l_{lp}+a).

6. PREDICTION OF SLIP

Using the finite difference method, the output solving the final differential numerically,are the axial and peeling forces in each node. From equation (1), it can be seen that the shear flow (q) is equal to the first derivative of the axial force in each node, hence the following formula can be used,

$$-q_n = \mathrm{N_{n-1}} - \mathrm{N_{n+1}} / 2.\Delta x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

where, (q_n)is the shear flow at node number (n), (N_{n-1}) and (N_{n+1}) are the axial force in nodes before and after node number (n) and (Ax) is the spacing between every successive nodes.When the value of shear at each node has been obtained, the value of slip at the same node can be defined using equation (41).

7. STRAIN AND STRESS

If we see equations (32),(33),(35) and (36) ,It must be established that the strain in each element is due to direct strain and bending strain, these equations give the strain for the three elements at

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the upper and lower interface of timber beam. By substituting the values of ($W_{a,xx}$), ($W_{b,xx}$)and ($W_{c,xx}$) from equations (15),(16) and (17) and replace the values of (Ma/B_1), (Mb/B_2) and (Mc/β_3) from equations (27),(28),(29) and (30) respectively. The values of strain at the three elements can be obtained as,

$$\varepsilon_1 = \frac{N}{\alpha_1} - \left[\frac{M - N(d_1 + d_2)}{B_2} + \frac{F1,xx}{Kn}\right] \cdot \frac{B_2}{B_1 + B_2} \cdot \bar{h}_1 \quad \ldots\ldots (64)$$

$$\varepsilon_2 = \left[\frac{M - N(d_1 + d_2)}{B_1} + \frac{F1,xx}{Kn}\right] \cdot \frac{B_1}{B_1 + B_2} \cdot h_2 - \frac{2N}{\alpha_2}$$

due to upper layer……………………………….……….…(65)

$$\bar{\varepsilon}_2 = \left[\frac{M - N(d_1 + d_2)}{B_3} + \frac{F2,xx}{Kn}\right] \cdot \frac{B_2}{B_2 + B_3} \cdot h_2 - \frac{2N}{\alpha_2}$$

due to lower layer ……………………......(66)

$$\varepsilon_3 = \frac{N}{\alpha_3} - \left[\frac{M - N(d_1 + d_2)}{B_2} + \frac{F2,xx}{Kn}\right] \cdot \frac{B_2}{B_2 + B_3} \cdot h_3 \quad \ldots\ldots (67)$$

Considering, equations (64, 65, and 66) and (67) and elastic material, the stress in each node can be found from hook’s low:

$$\sigma_1 = E_{up} \cdot \varepsilon_1 \ldots\ldots\ldots\ldots (68)$$

$$\sigma_2 = E_{co} \cdot \varepsilon_2 \ldots\ldots\ldots\ldots (69)$$

$$\bar{\sigma}_2 = E_{co} \cdot \bar{\varepsilon}_2 \ldots\ldots\ldots\ldots (70)$$

$$\sigma_3 = E_{lp} \cdot \varepsilon_3 \ldots\ldots\ldots\ldots (71)$$

8. PREDICTION OF DEFLECTION

By using one of the equations (18, 19) or *(*20*)* Deflection can be found together with equations (21) or (22) as below:

$$\frac{1}{Kn} \cdot \frac{d^4 W_b}{dx^4} + \frac{d^4 F_1}{dx^4} = -\frac{d^2 N_a}{dx^2} \cdot h_1 - \frac{(\rho + F_1)}{\beta_2} \quad \ldots\ldots (72)$$

9. NUMERICAL SOLUTION

we can determine (d^4F_1/dx^4) and (d^2N/dx^2) From equation (72), by applying the right stencils from finite difference method see Fig.(2) , for each derivative and for each case, and the equations up contain derivatives of fourth order in terms of F_1 and W which can be expressed in finite difference form using five nodes points as given below :

$$\frac{d^4 y}{dx^4} = \frac{1}{\lambda^4}\left(Y_{i-2} - 4.Y_{i-1} + 6.Y_i - 4.Y_{i+1} + Y_{i+2}\right) \quad \ldots\ldots\ldots\ldots (73)$$

Then the values that obtained above are known in each node, and equation (72) becomes with one derivative variable in fourth order of (W_b).Boundary Conditions While, there are only one unknown, (W_b), at each node, solution of the resulting set of algebraic equations requires specifying boundary conditions at each end of the layerd beam and since there are two external nodes at each end, then, boundary conditions are required at each end, as below:

1. $W_{b,xx} = M_b / B_2$ …………………………………... (74)

2. By derivative equation (16) once with respect to x, and equates with equation (8) then substituting for (S_b= TS) at (x= 0) the second boundary condition becomes:

$$W_{b'xxx} = S_b - ((d_1 + d3)/B_3).N_{c,xx} \quad \ldots\ldots\ldots\ldots (75)$$

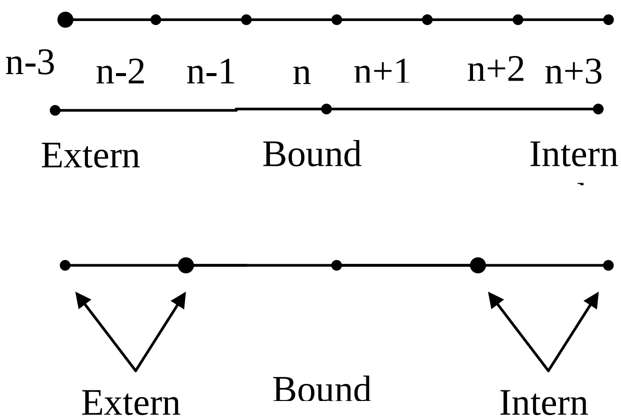

Fig. (2): Nodes for finite deference

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10. RESULTS AND VALIDATION

To develop a clear understanding of the problem of interlayer slip, the first series submitted by Goodman and consists of three wood layers with equal length components. Goodman takes a typical layered beam, which consists of three equal layers, and has the same mechanical properties, made of wood and each layer is connected to the other by nail (dimensions and other details are shown in Figure(3) and Table (1).

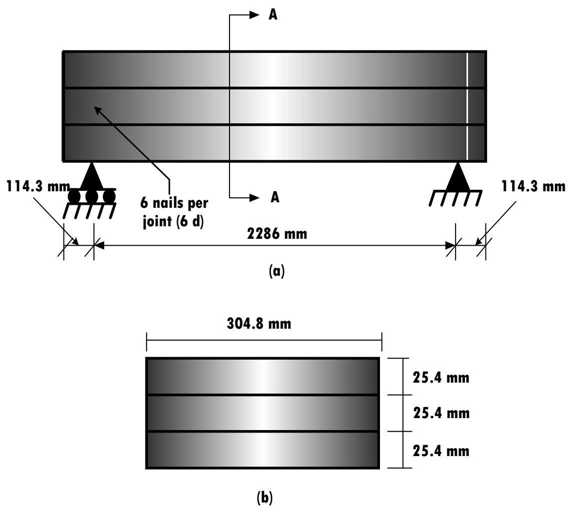

Fig. (3): (a) A Typical Layered Beam System**.**
(b) Section (A-A) at the Beam[2]

Good agreement between the experimental and theoretical results was obtained as shown in table (2) and figures (4) and (5); the large effect of the slip is evident when the actual deflection of the beam is compared with the one for equivalent solid beam. Comparison between current solution and Goodman's solution is carried out for central concentrated load in addition to a convergence study, as clarified in Table (2).

Table (1): Material Properties of Johnson's Example.

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Material	Property	Value
Wood layer	Total length (mm)	2286
	Modulus of Elasticity E_w (N/mm^2)	$5.8*10^3$
	Width (mm)	304.8
	Thickness (mm)	25.4
6 d. Nails	Connector Modulus k (N/mm)	$0.278*10^3$
	Diameter (mm)* Height (mm)	(19.5*50.8)
	Spacing (mm)	288.6

Table 2 Comparison between Solutions for the Suggested Models and Goodman's Solution.

Type of test		Experimental value (mm) from Goodman for slip	Theoretical value (mm) from suggested model for slip
			Numerical solution
Point load at L/2 (888.8N)	Max. slip (mm)	0.104	0.102
	Max. deflection (mm)	6.68	6.21
	Max. Axial force (KN)*10^4	6.85	6.65

Figure (4), shows variation of interface slip, for the same example, described previously central loading conditions are applied to the beam .

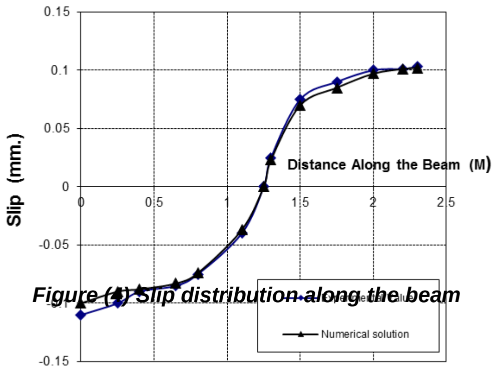

Figure (4) Slip distribution along the beam

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Fig. (4): Slip distribution along the beam

Finally figure (5) show variation of deflection , for the same example along the beam,

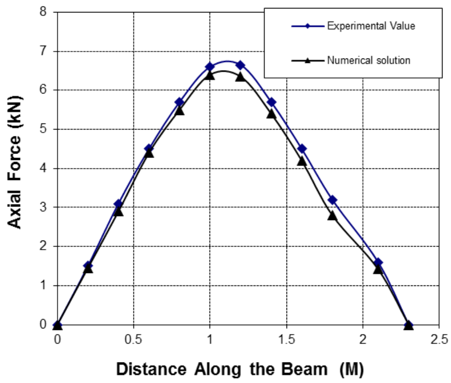
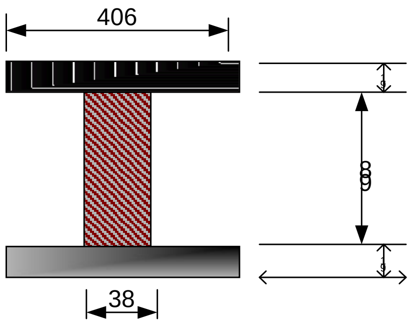


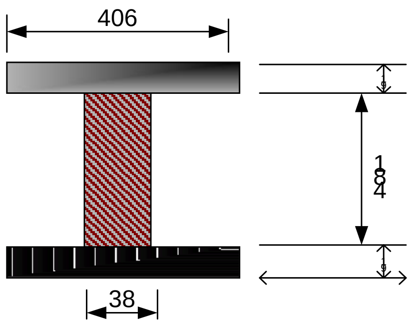
Fig. (5): Deflection distribution along the beam

McCutcheon presented a simple procedure to computing the composite stiffness of a wood bending member with sheathing attached onrigidly to both edges. The validity of the theory was checked by construction and testing (24) I-beams.

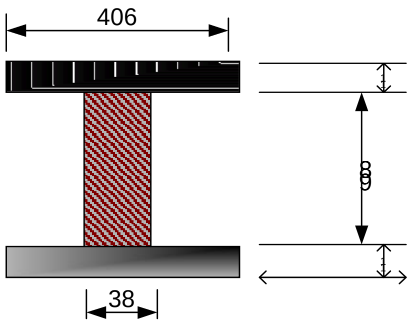
The beams were constructed from two sizes of No. 2 spruce –pine –fir webs [38 × 89mm× 2.44 m] and [38 × 184 mm × 2.44 m] and two types of flanges were employed [19-mm CDX plywood and 11-mm oriented strand board]. The flanges were all 406 mm wide and 8d common nails spaced at 152 mm. were used to fasten the flanges to the webs, see Figure (6).



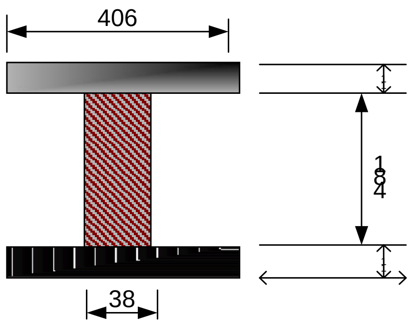
(a) I-Beam consists of spruce-pine-fir web and CDX plywood flanges -size 1 (All dimensions in mm)



(b) I-Beam consists of spruce-pine-fir web and CDX plywood flanges -size 2 (All dimensions in mm)



(c) I-Beam consists of spruce-pine-fir web and oriented strandboard flanges -size 1 (All dimensions in mm)



(d) I-Beam consists of spruce-pine-fir web and oriented strandboard flanges -size 2 (All dimensions in mm)

Fig. (6): Types of I-Beams used in the tests

Each I-beam was tested three times: first with both flanges continuous and slip measured at points A and B (Figure 7); second with the bottom flange cut and slip measurements also at C ; third with the top flange also cut and slip measured at D.

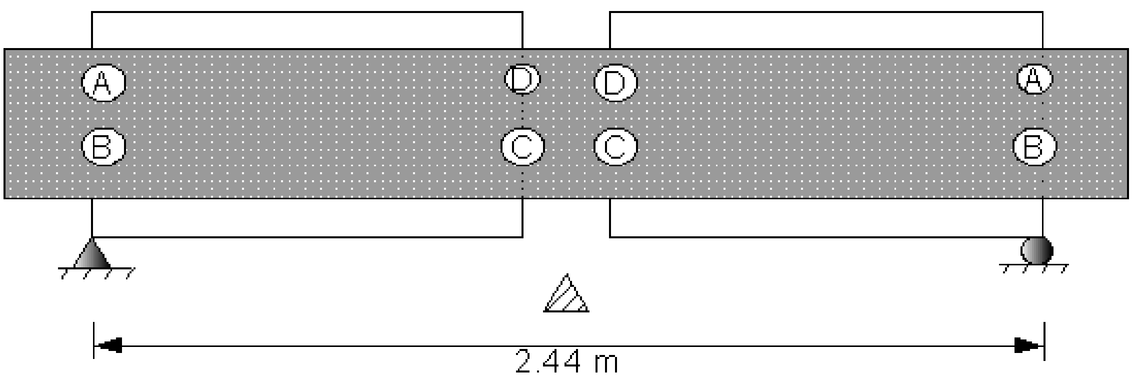


Fig. (7): Beam deflection and slip (A,B,C,D) measurements

The results of the I- beam tests, shown in Table (3) are reported as load/deflection ratios for the composite beam stiffnesses, and load/slip ratios for the interlayer slip measurements.

The I-beams with continuous flanges were considerably stiffer than the webs alone. In general, the results gave very good estimates of composite beam stiffness.

Table (3): comparison between Top slip obtained by McCutchoen and the solutions submitted by suggested models for 89 mm I-beam test results.

Flange type	I. Web		Top load/ slip		
(1)	E (10 6 kpa) (2)	Load/deflection (N/mm) (3)	Test (N/mm.) (4)	numerical (N/mm.) (5)	Difference (%) (7)
PLY-PLY	11.02	145.49	3560.50	4077.06	9.6
			3215.53	3624.85	-4.7
			4756.27	5421.18	-4.7
	11.51	151.93	3950.15	4175.37	8.2
			3374.61	3719.58	1.8
			5397.95	5616.01	-3.3
	9.30	122.80	3344.23	3730.30	7.0
			2911.67	3285.24	1.8
			4604.34	4724.10	6.2
PLY-OSB	7.10	93.66	2379.03	3179.78	11.8
			2060.87	2840.18	11.5
			3197.66	3810.74	9.6
	11.51	151.93	3371.04	4064.55	-12.6

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			2868.78	3707.07	0.0
			4911.78	5590.99	0.0
	7.65	101.00	3099.35	3290.60	0.2
			2618.54	2947.42	0.1
			3998.41	4032.37	0.0
OSB-PLY	8.13	107.42	2373.67	3315.63	48.9
			2211.01	2874.14	9.8
			3816.10	4239.71	5.7
	9.85	130.12	3869.72	3664.17	12.6
			3088.63	3215.53	13.2
			4264.74	4935.01	14.4
	12.67	167.48	3846.48	4237.93	24.5
			3419.30	3778.56	12.9
			5819.77	6077.16	7.9
OSB-OSB	10.34	136.56	3683.83	3653.45	5.6
			3208.38	3297.75	11.1
			4833.13	5106.60	8.7
	11.65	153.71	4084.21	3917.98	5.0
			3546.20	3558.71	16.7
			4663.33	5635.67	10.1
	9.22	130.02	3547.99	3571.23	9.0
			2954.57	3215.53	-1.7
			3882.23	4940.37	13.6

First line – continuos flanges ; second line – bottom flange cut at midspan ; third line –top flange also cut at midspan

11. CONCLUSION

A finite difference model was presented to study the behavior of glued laminated timber beam. The current model provides information on the slip, separation and stresses at each node that divided on it. Results from the analytic help to predict the slip, deflection, and stresses at each node. It is concluded, that basic understanding of the interlayer slip mechanics and the solution methods applicable to this problem have been gained. It is assumed that this will lead to considerable improvement in rational design procedure for layered beam systems. Finally, when comparison is made between test results available from literature and the predicted results presented in this study, a close agreement between these results is concluded.

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