# DETERMINATION OF COMPETITION FACTOR FOR TREE GROWTH WITHIN A STAND 

Tarie K. SALIH* ${ }^{*}$ MUZAhim Y. Saied ${ }^{* *}$ and SALIH T. WALI ${ }^{*}$<br>*Dept. of Forestry, College of Agricultural Engineering Sciences, University of Duhok, Kurdistan Region-Iraq.<br>${ }^{* *}$ Dept. of Forestry, College of Agriculture and Forestry, University of Mosul-Iraq.

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#### Abstract

This study has two purposes; the first one is development of a new method in determining a competition factor for Pinus brutia trees grown in a stand, and the second one is development of a theoretical method for determination potential growth of individual trees within a stand. This potential growth can be used to estimate the growth of a tree as if it grows in a competition free area. Competition between trees has a strong effect on growth potential. Dense forests need thinning to ensure providing the essential growth requirements. The developed factor can be used to give the answer for when and how much of stocking should be removed. Two samples of trees were selected, one from an open grown area and the other from forest stands. For each sample many regression equations were developed, for regressing, the diameter growth with different forms of diameter. The developed equations were undergone many tests of precision, to determine the one that best fits the dataset. At last the equation, $\mathrm{Dg}=6.60171-0.753194 * \sqrt{\mathrm{D}}$, and equation $\mathrm{Dg}=7.19547-0.792438 * \sqrt{\mathrm{D}}$ were finally selected for regressing the diameter growth with diameter at breast height for trees grown in a stand and open grown trees respectively. These selected equations were used to calculate the competition factor. Furthermore the equation; $C W=4.73265 * \exp ^{(0.0157654 * D)}$ was finally selected among 23 others to be used for calculating the growth potential of trees grown within a stand.


KEYWORDS: Completion factor, Competition indices, Diameter growth, Individual tree growth, potential growth
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## INTRODUCTION

The growth of a tree is defined as, the increase in its dimensions under a specific period, while diameter growth is the increase of diameter of a tree in $\mathrm{cm} \mathrm{y}^{-1}$. (Vanclay, 1994; Husch, et al., 2003; Pretzsch, 2009; von Oheimb, 2011) Competition between trees is one of the most important factors that have a significant effect on diameter growth of trees. (Lorimer, 1983; Pukkal, 1987; Schröder, 2002; Kaitaniemi, et al., 2010). The competition is closely related to stand density. However, competition is generally used with individual trees, while density is used in stand level (Avery et al., 2015).

The potential growth of a site should be utilized as much as possible to attain its maximum growth capacity. In establishing a stand, seedlings are usually planted very close to each other, because of their limited need for water, light, nutrients, and space. As they grow in size, these requirements increase and hence the competition may start between trees. The result might lead to domination of large or healthy
trees over some other trees. In addition to the natural pruning of branches (that don't get sufficient light) may occur, many suppressed trees, may die due to intensive competition. In order to avoid or decrease such natural phenomenon, foresters and forest owners have to make a thinning program for harvesting of some individual trees. (Nebeker, et al., 1985; Soucy et al., 2012) Thinning has multiple benefits; the wood of the harvested trees may be used for different purposes depending on their sizes, qualities and species. Removing the unhealthy trees from the stand will protect the heathy trees from insect's attacks and diseases. In addition to this, it will provide more spaces for the remaining trees, that enhance their growth especially diameter growth. (Zhang, et al., 1997; Kim, et al., 2016). Thinning in sapling stands can increase diameter growth and improve species composition of trees in the main canopy. Recent studies have shown that crown growing space have positive effect on diameter growth and negative effect on height growth of trees. Harrington, et al., (1983), stated that, the diameter growth models developed in
this study can be used for different purposes, including prediction of future growth under current and future projected conditions. They can be used to estimate the volume growth of the trees, using volume tables. Furthermore, these models have the ability of reducing the uncertainties of growth and yield predictions (Ashraf et al., 2015)

Although, a huge number of studies and investigations have been made in different parts of the world in order to find the competition factor or other measures of stand density, yet, none of these is entirely satisfactory. Maleki et al., (2015), tested both spatially and non - spatially sets of competition indices to quantify the effects of neighboring trees on diameter growth of silver birch trees. Rivas, et al., (2005) studied the effect of competition on individual tree basal area growth in mature stands of Pinus cooperi in Durango (Mexico). (von Oheimb, et al., 2011; Pukkala et al., 1987) studied the effects competition on the radial growth of individual trees. Schröder, et al., (1999) used the basal area of large trees to develop a distance dependent competition index for Maritime pine trees in northwestern Spain. Generally speaking competition indices, can be grouped in three different categories; 1. Indices, that, use the zone of influence among the neighboring trees (Newnham, 1964; Opie, 1968; Gerard, 1969; Ek and Monserad 1974; Uriarte, et al., 2004). 2. Indices using growing space that determine the potentially available area for growing trees (Brown 1965; Adland 1974) 3. Indices, which use both relative diameters and distances between subject trees and their competitors. (Hamilton, 1969; Biging, et al., 1995) used distance independent competition measures to explain the height growth variation and diameter squared variation of individual conifer trees. Bella, (1971) developed a model which represents mathematically competitive interaction between individual trees. His model was consisted of both size of trees and the distances between them. Contreras, et al., (2011) used tree competition indices as predictors of basal area increment in western Montana forests.

In studying the relationships between tree growth and diameter Enquist et al. (1999) suggested that diameter growth will follow the form of a power function with an exponent of $1 / 3$, this theory is far from commonly accepted (Muller-Landau et al., 2006).

The developed measure of competition of the present study may be used as a powerful tool in giving answer to when and where thinning should be
practiced, as well as a predictor variable in growth and yield modeling.

This study has two main objectives; the first one is determining of competition index using empirical method while, the second one regarded with potential growth of an individual tree using both available spaces between the tree and its neighbors as well as the ratio between their size.

## MATERIALS AND METHODS

Two different methods were used to develop competition factor in this study:

## Empirical Method

In this method two groups of Pinus brutia ten. grown, in Duhok governorate, were selected. The first group consisted of 60 trees with proper stem quality and high vigor conditions, were taken from an open grown area, in other words such trees which were free from competition with neighboring trees, while the second group of trees consisted of 75 stand grown trees (planted originally at a distance of $3 * 4 \mathrm{~m}$ ) Table (1) Increment borer was used to extract cores from all trees of both groups. From these cores diameter and diameter growth data were obtained. Cores were treated with sandpaper in order to obtain a more conspicuous view for annual rings. The cores were installed on a wooden basement for imaging purpose, which were done with a high resolution camera (Nikon Digital Camera D3100). (Figure1).

Fig 1. shows an image of an extracted core. These images were transferred to computer to be used for measuring and calibration the annual ring using a CDendro 9.0.1. software program along with Cybis coordinate Recorder, (Salih et al., 2019)

For each group of trees, many regression equations were developed to relate periodic diameter growth as dependent variable with different forms of initial diameter as independent variables. The relative efficiency of the developed equations of each group was undergone several tests of precision in order to select the most appropriate one to be used for calculating competition Factor, according to the following formula:
Competition factor $=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Dgc}_{\mathrm{i}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Dg}}$
Where: Dgc is the diameter growth estimation from the selected equation for trees grown under competition stress and Dg is the diameter growth estimation from equation for open growth trees

Table (1): Statistical description of the collected data, for determination of competition factor for empirical method


Fig.( 1): Shows how the cores were installed for photographing, as well as calibrating, and measuring the annual ring
The procedure of the whole process from core extraction to statistical analysis is given in the following figure:


Fig. (2): The whole procedure from data collection to statistical analysis and development of equations

## Theoretical Method

This method is based on two principles:

1. The first one is the space occupied by any tree is proportional to its size. A small tree requires lesser growing space than what a large one needs (Hegyi, 1974).
2. The second one is the competition between a tree and its neighbors are proportional to the ratio between their diameters. Therefore, the occupied space by each one is determined by its size (Dimeter, height or volume).

Excel tool, Statgraphic and SAS packages were used for data processing and equation
development. The diameter growth in its different forms was used as a dependent variable, including linear and nonlinear equations and different forms of diameter at breast height as independent variables. (Uriarte et al., 2004; Canham et al., 2006; Coates et al., 2009; Stadt et al., 2007; Boivin et al., 2010; Berger et al., 2000)

## RESULTS AND DISCUSSION

## The Result of the Empirical Method:

This method consists of development of equations from dataset collected from two different sites:
a. Open grown area, which are free from competition.
b. Stand under competition stress (trees are competing with each other).

## Development of Regression Equations for the Open Grown Trees:

Using the above mentioned tools and facilities, (23) regression equations were developed along with some measures of precision, for the first group. See (Table 2), which shows the developed regression equations for open grown trees in Duhok Governorate.

Table (2): List of developed equations for diameter growth

| No | Equations |  | Adj-R | RMSE | MAE |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Original Form Equation |  |  |  |
| $\mathbf{1}$ | $\mathrm{Dg}=5.30961-0.0786297^{*} \mathrm{D}$ | 66.1395 | 0.65903 | 0.52636 | 1.35554 |
| $\mathbf{2}$ | $\mathrm{Dg}=\left(2.35631-0.0221606^{*} \mathrm{D}\right)^{2}$ | 68.87 | 0.17461 | 0.13882 | 1.39213 |
| $\mathbf{3}$ | $\mathrm{Dg}=7.19547-0.792438^{*} \sqrt{\mathrm{D}}$ | 69.118 | 0.62938 | 0.48845 | 1.41117 |
| $\mathbf{4}$ | $\mathrm{Dg}=8.82717-1.77176^{*} \ln (\mathrm{D})$ | 68.2142 | 0.63852 | 0.49736 | 1.40393 |
| $\mathbf{5}$ | $\mathrm{Dg}=2.04765+24.042 / \mathrm{D}$ | 55.5724 | 0.75489 | 0.61398 | 1.39842 |
| $\mathbf{6}$ | $\mathrm{Dg}=6.03028-0.139249^{*} \mathrm{D}+0.00105212^{*} \mathrm{D}^{2}$ | 68.4135 | 0.63651 | 0.48105 | 1.42752 |
| $\mathbf{7}$ | $\mathrm{Dg}=6.16224^{*} \operatorname{Exp}\left(-0.0261491^{*} \mathrm{D}\right)$ | 69.1286 | 0.62927 | 0.48149 | 1.42551 |
| $\mathbf{8}$ | $\mathrm{Dg}=8.7803^{*}\left(1-0.207601^{*} \mathrm{D}^{\wedge} 0.347453\right)$ | 68.7695 | 0.63292 | 0.48182 | 1.41382 |

Logarithmic Equations

| $\mathbf{9}$ | $\operatorname{Ln}(\mathrm{Dg})=\left(1.7894-0.0256575^{*} \mathrm{D}\right)$ | 70.7961 | 0.19319 | 0.15181 | 1.45682 |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ | $\operatorname{Ln}(\mathrm{Dg})=\left(2.36661-0.250858^{*} \sqrt{D}\right)$ | 69.5256 | 0.19735 | 0.15334 | 1.47873 |
| $\mathbf{1 1}$ | $\operatorname{Ln}(\mathrm{Dg})=\left(2.82834-0.543378^{*} \ln (\mathrm{D})\right)$ | 64.2956 | 0.21361 | 0.16998 | 1.46036 |
| $\mathbf{1 2}$ | $\operatorname{Ln}(\mathrm{Dg})=2.82834-0.543378^{*} \log (\mathrm{D})$ | 64.2956 | 0.21361 | 0.16998 | 1.46036 |
| $\mathbf{1 3}$ | $\operatorname{Ln}(\mathrm{Dg})=\left(1.46019-0.000415217^{*} \mathrm{D}^{\wedge} 2\right)$ | 65.73 | 0.20928 | 0.17324 | 1.30186 |

Y-Reciprocal Equations

| $\mathbf{1 4}$ | $1 /(\mathrm{Dg})=1 /\left(0.103871+0.0093593^{*} \mathrm{D}\right)$ | 70.4762 | 0.0710153 | 0.053999 | 1.62061 |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $\mathbf{1 5}$ | $1 /(\mathrm{Dg})=1 /\left(-0.0926544+0.0886701^{*} \sqrt{\mathrm{D}}\right)$ | 64.8804 | 0.07745 | 0.06112 | 1.58884 |
| $\mathbf{1 6}$ | $1 /(\mathrm{Dg})=1 /(-0.235857+0.185679 * \ln (\mathrm{D}))$ | 61.9873 | 0.08673 | 0.06845 | 1.55523 |


| 17 | $1 /(\mathrm{Dg})=1 /\left(0.217276+0.000159781 * \mathrm{D}^{2}\right)$ | 73.0118 | 0.06789 | 0.05439 | 1.55997 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | $1 /(\mathrm{Dg})=0.187235+0.00234706 * \mathrm{D}+0.000121705^{*} \mathrm{D}^{\wedge} 2$ | 72.8401 | 0.06811 | 0.05286 | 1.59525 |
| 19 | $1 /(\mathrm{Dg})=\left(0.127842+0.00532593 * \mathrm{D}^{\wedge} 1.13445\right)$ | 70.8901 | 0.07051 | 0.05229 | 1.62149 |
| $\mathbf{Y}$ - Square Equations |  |  |  |  |  |
| 20 | $\mathrm{Dg}^{2}=(25.7629-0.532413 * \mathrm{D})$ | 59.7551 | 5.11131 | 4.07659 | 1.3526 |
| 21 | $\mathrm{Dg}^{2}=\left(39.2642-5.51378^{*} \sqrt{\mathrm{D}}\right)$ | 66.044 | 4.69499 | 3.70525 | 1.44376 |
| 22 | $\mathrm{Dg}^{2}=\left(51.709-12.6764^{*} \ln (\mathrm{D})\right.$ ) | 69.0155 | 4.4848 | 3.37937 | 1.46274 |
| 23 | $D g^{2}=(2.74338+181.004 / \mathrm{D})$ | 62.4456 | 4.9375 | 4.00006 | 1.41008 |

Where: Dg is the diameter growth and D is the diameter at breast height (DBH).

Although so many equations were developed, but actually, only one of them which, is the most appropriate one that fits our dataset is needed. Therefore the developed equations were undergone several tests of precision in order to find the most suitable one. As it can be seen in the table above that the fit statistics used in this study to determine how well the regression function fit the sample data are:

Adjusted coefficient of determination (Adj$\mathrm{R}^{2}$ ), Root of Mean Square Error (RMSE), Mean Absolute Error (MAE), and Durbin Watson Test (D-W). It is not possible to test the precision of different equations unless their dependent variables are of the same form (Furnival, 1961) .Therefore, the precision of equations within each group was tested with each other in order to select the best candidate. Then the selected equations were tested using Furnival index or Ohtomos test of unbiasedness.(Ohtomo 1956)

At the beginning (Adj-R2), (RMSE), (MAE), and (D-W) were used to select one equation from each group. The range of ( $\mathrm{Adj}-\mathrm{R}^{2}$ is 0 to 1 ). The closer the value of this measure to exact one, the better is the equation. The precision of an equation increases as the value of RMSE, MAE, decreases. D-W is used to see if there is an autocorrelation between residuals of any observation and the proceeding one, for the tested equation. The following formula is used to calculate the last measure of precision (Neter et al., 1996)

$$
\mathrm{D}=\frac{\sum_{1}^{\mathrm{n}}\left(\mathrm{e}_{\mathrm{i}}-\mathrm{e}_{\mathrm{i}-1}\right)^{2}}{\sum_{1}^{\mathrm{n}} \mathrm{e}^{2}{ }_{\mathrm{i}}} \ldots \ldots \ldots \text { (2) }
$$

Where (D) is calculated value of D-W and ( $\mathrm{e}_{\mathrm{i}}$ and $\mathrm{e}_{\mathrm{i}-1}$ ) is the difference between the residual of the $\mathrm{i}^{\text {th }}$ observation and the previous one

The value of D-W ranges between ( 0 and 4).

| D-W $=0$ | D-W $=2$ | D-W $=4$ |
| :--- | :--- | :--- |
| + Autocorrelation | No | - |

As a thumb rule, there is no evidence of autocorrelation, if the value of D-W lies between (1.5 and 2.5) but for more accurate determination, D-W value was calculated for each equation, and compared with the tabulated value ( which depends on the degrees of freedom) to determine if there is autocorrelation or not.

Taking these measures of precision in consideration, the equations number $3,9,17$ and 22 was selected from the first group, second group, third group, and fourth group, respectively. (Table 2)

These selected equations were undergone another test of precision called Ohtomo's unbiased test. This test can be summarized as follow:

The estimated values of the dependent variable, from each candidate are regressed on the corresponded actual values of the dependent variable in a simple regression equation, as follow:

$$
\hat{y}_{i}=b_{0}+b_{1} y_{i} \ldots \ldots \ldots . \text { (3) }
$$

Where: $\left(\hat{y}_{i}\right.$ and $\left.y_{i}\right)$ are the estimated and actual values of $\mathrm{i}^{\text {th }}$ observation respectively and ( $b_{0}$ and $b_{1}$ ) are y-intercept and regressions coefficient of the equation respectively. It is well known that the best candidate is that, which has
the estimated values of $y$ very close to the corresponding values of actual values (y). This
case meets if $b_{0}$ and $b_{1}$ close to zero and one respectively.
(Table

Table (3): Estimated values of Ohtomo's test for the selected candidates

| Equation | $\mathbf{b}_{\mathbf{0}}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{R}^{2}$ | $\mathbf{D - W}$ | $\mathbf{b}_{\mathbf{0}} \mathbf{+}\left\|\mathbf{1}-\mathbf{b}_{\mathbf{1}}\right\|$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\hat{\mathbf{y}}_{\mathbf{3}}=\mathbf{b}_{\mathbf{0}}+\mathbf{b}_{\mathbf{1}} \mathbf{y}$ | 0.9953 | 0.6964 | 69.12 | 1.54 | 1.299 |
| $\hat{\mathbf{y}}_{\mathbf{9}}=\mathbf{b}_{\mathbf{0}}+\mathbf{b}_{\mathbf{1}} \mathbf{y}$ |  |  |  |  |  |
| $\hat{\mathbf{y}}_{\mathbf{1 7}}=\mathbf{b}_{\mathbf{0}}+\mathbf{b}_{\mathbf{1}} \mathbf{y}$ | 1.0091 | 0.6741 | 69.12 | 1.53 | 1.335 |
| $\hat{\mathbf{y}}_{22}=\mathbf{b}_{\mathbf{0}}+\mathbf{b}_{\mathbf{1}} \mathbf{y}$ | 1.2254 | 0.5824 | 66.83 | 1.65 | 1.643 |

Where: $\hat{\mathrm{y}}_{3}, \hat{\mathrm{y}}_{9}, \hat{\mathrm{y}}_{17}$, and $\hat{\mathrm{y}}_{22}$, are estimated values of diameter growth from equations $3,9,17$ and 22 respectively.

According to Ohtomo's test, the best equation is the one which has the values of $b_{0}$ and $b_{1}$ are very close to zero and one respectively, therefore a new index was developed, in order to take both of them in consideration at the same time. This index is $b_{0}+\left|1-b_{1}\right|$. (Salih et al., 2019) According to this index, the equation, which has lowest value, is statistically the most precise one. Accordingly, the equation number 22 is superior to the rest of equation, because of having the lowest value. On the other hand, on the mean while the third equation is the best one, based on, the values of $\mathrm{R}^{2}$ and (D-W) (Table 3). Making a
balance between these two equations and having focus on the simplicity of these candidates in application, the third equation, $\mathrm{Dg}=7.19547$ $0.792438 \sqrt{D}$ was finally selected as the best one Development of Equations for Stand Grown Trees

A procedure analogous to the one used for the first group was followed. 75 observations were used for developing of 21 regression equations along with different measures of precision (Table 4).

Table (4): List of the developed equations for stand grown trees.

| No | Equations | Adj- $\mathbf{R}^{2}$ | RMSE | MAE | D-W |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Original Form Equations |  |  |  |  |  |
| 1 | $\mathrm{Dg}=5.19487-0.0921618 * D$ | 65.3272 | 0.653678 | 0.532384 | 1.59558 |
| 2 | $\mathrm{Dg}=6.60171-0.753194 * \sqrt{D}$ | 66.3723 | 0.643751 | 0.515326 | 1.62913 |
| 3 | $\mathrm{Dg}=7.32447-1.39702^{*} \ln (\mathrm{D})$ | 64.402 | 0.662342 | 0.516423 | 1.62645 |
| 4 | $\mathrm{Dg}=5.65174-0.155639 * \mathrm{D}+0.00163959 * \mathrm{D}^{2}$ | 66.833 | 0.639326 | 0.501364 | 1.63003 |
| 5 | $\mathrm{Dg}=6.23498 *\left(1-0.0853766 * \mathrm{D}^{\wedge} 0.575037\right)$ | 65.9371 | 0.647904 | 0.516508 | 1.62549 |
| 6 | $\mathrm{Dg}=6.17854 *\left(1-0.0553117^{*} \mathrm{D}^{\wedge} 0.641039\right)^{\wedge} 1.36542$ | 65.5982 | 0.651119 | 0.51375 | 1.62953 |
| 7 | $\mathrm{Dg}=14.6362 /\left(1+1.68155^{*} \operatorname{Exp}\left(0.0364227{ }^{*} \mathrm{D}\right)\right.$ ) | 66.4326 | 0.643174 | 0.511931 | 1.623 |
| Logarithmic Equations |  |  |  |  |  |
| 8 | $\operatorname{Ln}(\mathrm{Dg})=(1.69975-0.0270529 * \mathrm{D})$ | 66.9617 | 0.185048 | 0.149897 | 1.63415 |
| 9 | $\operatorname{Ln}(\mathrm{Dg})=\left(2.10694-0.219672^{*} \sqrt{D}\right)$ | 67.145 | 0.184534 | 0.147434 | 1.65506 |


| 10 | $\operatorname{Ln}(\mathrm{Dg})=\left(2.31082-0.404898{ }^{*} \ln (\mathrm{D})\right.$ ) | 64.3225 | 0.192297 | 0.150551 | 1.64706 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | $\mathrm{Ln}(\mathrm{Dg})=9.11381 * \mathrm{D}^{\wedge}-0.358345$ | 61.2257 | 0.69126 | 0.537191 | 1.59042 |
| 12 | $\operatorname{Ln}(\mathrm{Dg})=5.60784^{*} \operatorname{Exp}\left(-0.027557^{*} \mathrm{D}\right)$ | 66.9341 | 0.638351 | 0.509618 | 1.63095 |
| 13 | $\operatorname{Ln}(\mathrm{Dg})=\left(0.602462+0.00234504^{*} \mathrm{D}^{\wedge} 1.59192\right)^{\wedge}-1$ | 67.9005 | 0.1824 | 0.145206 | 1.63921 |
| Y - Reciprocal Equations |  |  |  |  |  |
| 14 | $1 / \mathrm{Dg}=(0.161992+0.00838493 * \mathrm{D})$ | 65.624 | 0.0590856 | 0.0458217 | 1.69906 |
| 15 | $1 / \mathrm{Dg}=(0.0377876+0.0675943 * \sqrt{D})$ | 64.8361 | 0.0597589 | 0.0460162 | 1.70588 |
| 16 | $1 / \mathrm{Dg}=\left(-0.0225673+0.123711^{*} \ln (\mathrm{D})\right)$ | 61.2167 | 0.0627591 | 0.0485961 | 1.69367 |
| 17 | $1 / \mathrm{Dg}=0.145809+0.0106334 * D-0.0000580765^{*} \mathrm{D}^{2}$ | 65.4491 | 0.0592357 | 0.0452416 | 1.69885 |
| Y - Square Equations |  |  |  |  |  |
| 18 | $\left(D_{g}\right)^{2}=(25.6151-0.660903 *$ D $)$ | 61.6742 | 5.06913 | 4.05529 | 1.58758 |
| 19 | $\left(D_{g}\right)^{2}=(35.8248-5.431 * \sqrt{D})$ | 63.3695 | 4.95574 | 3.89871 | 1.62836 |
| 20 | $\left(D_{g}\right)^{2}=(41.1834-10.1276 * \ln (\mathrm{D}))$ | 62.1655 | 5.03653 | 3.84508 | 1.62931 |
| Square Root Equations |  |  |  |  |  |
| 21 | $\sqrt{\text { D }}=(2.30367-0.0248011 *$ D $)$ | 66.4631 | 0.171548 | 0.13935 | 1.6109 |

Where: Dg is the diameter growth and d is diameter at breast height.

The dependent variable in these equations appeared in five different forms including original, logarithmic, y- reciprocal, y-square form, $y$-square root. The same technique and methodology mentioned before were for both data processing, equation developing and screening of equations for selecting the most appropriate one were used and accordingly the second equation appeared in the table 4 was selected as the best one which fits our dataset. $(D g=6.60171-0.753194 * \sqrt{D})$. Fortunately it was compatible to the other selected equation and hence easier for interpretation and conclusion. This equation was used with the selected equation from the former group (the
third equation from the table 2 ) for computing the competition index, using the formula (1)

## Calculation of the Competition Index

The competition factor was calculated according the following proposed formula:

$$
\begin{equation*}
\text { Competition factor }=\frac{\sum_{i=1}^{\mathrm{n}} \mathrm{Dgc}_{\mathrm{i}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Dg}_{\mathrm{i}}} . \tag{1}
\end{equation*}
$$

This means that, the two equation previously selected were used in calculating of the competition factor. In order to do this, a group of trees with d.b.h ranged from 4 cm to 51 cm were substituted in the two selected equations, and these values were summed and substituted in the above mentioned formula to get the Competition factor. Table 5 shows both diameter and estimated diameter growth of the first group.

Table (5): The estimated values of diameter growth from the selected equation belonging to the open grown trees

| $\mathbf{D}(\mathbf{c m})$ | $\mathbf{D g}(\mathbf{c m})$ | $\mathbf{D}(\mathbf{c m})$ | $\mathbf{D g}(\mathbf{c m})$ | $\mathbf{D}(\mathbf{c m})$ | $\mathbf{D g}(\mathbf{c m})$ | $\mathbf{D}(\mathrm{cm})$ | $\mathbf{D g}(\mathbf{c m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5.61 | 16 | 4.03 | 28 | 3 | 40 | 2.18 |
| 5 | 5.42 | 17 | 3.93 | 29 | 2.93 | 41 | 2.12 |
| 6 | 5.25 | 18 | 3.83 | 30 | 2.86 | 42 | 2.06 |
| 7 | 5.1 | 19 | 3.74 | 31 | 2.78 | 43 | 2 |
| 8 | 5.96 | 20 | 3.65 | 32 | 2.71 | 44 | 1.94 |


| 9 | 4.82 | 21 | 3.56 | 33 | 2.64 | 45 | 1.88 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 4.69 | 22 | 3.48 | 34 | 2.57 | 46 | 1.82 |
| 11 | 4.57 | 23 | 3.4 | 35 | 2.51 | 47 | 1.76 |
| 12 | 4.45 | 24 | 3.31 | 36 | 2.44 | 48 | 1.7 |
| 13 | 4.34 | 25 | 3.23 | 37 | 2.38 | 49 | 1.65 |
| 14 | 4.23 | 26 | 3.15 | 38 | 2.31 | 50 | 1.59 |
| 15 | 4.13 | 27 | 3.08 | 2.25 | 51 | 1.54 |  |

Table (6): The estimated values of diameter growth from the selected equation belonging to the stand grown trees

| D (cm) | DCg (cm) | D (cm) | DCg (cm) | D (cm) | DCg (cm) | D (cm) | DCg (cm) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5.10 | 16 | 3.59 | 28 | 2.62 | 40 | 1.84 |
| 5 | 4.92 | 17 | 3.50 | 29 | 2.55 | 41 | 1.78 |
| 6 | 4.76 | 18 | 3.41 | 30 | 6.60 | 42 | 1.72 |
| 7 | 4.61 | 19 | 3.32 | 31 | 2.41 | 43 | 1.66 |
| 8 | 4.47 | 20 | 3.23 | 32 | 2.34 | 44 | 1.61 |
| 9 | 4.34 | 21 | 3.15 | 33 | 2.27 | 45 | 1.55 |
| 10 | 4.22 | 22 | 3.07 | 34 | 2.21 | 46 | 1.49 |
| 11 | 4.10 | 23 | 2.99 | 35 | 2.15 | 47 | 1.44 |
| 12 | 4.00 | 24 | 2.91 | 36 | 2.08 | 48 | 1.38 |
| 13 | 3.89 | 25 | 2.83 | 37 | 2.02 | 49 | 1.33 |
| 14 | 3.78 | 26 | 2.76 | 38 | 1.96 | 50 | 1.28 |
| 15 | 3.68 | 27 | 2.69 | 39 | 1.90 | 51 | 1.23 |

$$
\sum_{i=1}^{n} \mathrm{DCg}=138.74 \text { and } \sum_{\substack{i=1 \\ n}}^{n} \mathrm{Dg}=154.58
$$

Competition Index $=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Dgc}_{\mathrm{i}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Dg}_{\mathrm{i}}}=\frac{138.74}{154.58}=$
0.90

One of the advantages of this method is that it is directly correlated to the growth. It is an index for the stand.

From the above mentioned calculation, one can conclude that the competition between our stand trees had caused a reduction of diameter growth with $10 \%$, as compared with open grown trees. This can be interpreted that these trees need thinning. This can be done by removing of about $10 \%$ of trees to give extra space to the remaining trees to grow better. It is very important to be careful in determination of the trees that will be removed. As a general rule, the weak, suppressed and unhealthy trees are the best candidates for removing. To do such task, the
experts from many fields of forestry may participate.

## The Development of the Theoretical formula

The principles that have been used here are already given in material and methods.

If it is desired to study the actual space occupied by a subject tree with a diameter of Dx, surrounded by trees $\mathrm{Dt}_{1}, \mathrm{Dt}_{2}, \mathrm{Dt}_{3}, \ldots, \mathrm{Dtn}$, which are situated at distances of $D_{1}, D_{2}, D_{3} \ldots \ldots . D_{n}$ respectively, we have to make some mathematical calculation as follow:

The length of the available distance between subject tree and the first tree $\left(\mathrm{R}_{1}\right)$
$\mathrm{R}_{1}=\frac{\mathrm{Dx}}{\mathrm{Dt}_{1}} * \mathrm{~d}_{1} * \frac{1}{2}$.
And the length of the available space between the subject tree and second tree $\left(R_{2}\right)$
$\mathrm{R}_{2}=\frac{\mathrm{Dx}}{\mathrm{Dt}_{1}} * \mathrm{~d}_{2} * \frac{1}{2}$.
In the same manner, the distance of the available space between the subject tree and other trees will be calculated.
$\mathrm{R}_{\mathrm{n}}=\frac{\mathrm{Dx}}{\mathrm{Dt}_{\mathrm{n}}} * \mathrm{~d}_{\mathrm{n}} * \frac{1}{2}$.
The arithmetic mean of the distances between the subject tree and the surroundings $(\overline{\mathrm{R}})$ can be calculated as follow:
$\overline{\mathrm{R}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Ri}}{\mathrm{n}}$
$\overline{\mathrm{R}}=\frac{\frac{1}{\frac{\mathrm{Dx}}{2}}\left(\frac{\mathrm{Dx}}{\mathrm{Dt}} \mathrm{t}_{1}\right)+\frac{1}{2}\left(\frac{\mathrm{Dx}}{\mathrm{Dt}_{2}} * \mathrm{~d}_{2}\right)+\ldots \frac{1}{2}\left(\frac{\mathrm{Dx}}{\mathrm{Dt}_{\mathrm{n}}} * \mathrm{~d}_{\mathrm{n}}\right)}{\mathrm{n}}$
$\bar{R}=\frac{D x\left(\frac{d_{1}}{D t_{1}}+\frac{d_{2}}{D t_{2}}+\frac{d_{3}}{D t_{3}}+\cdots \frac{d_{n}}{D t n}\right)}{2 n}$.
$\overline{\mathrm{R}}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{d}_{\mathrm{i}}}{\mathrm{Dt}_{\mathrm{i}}}$.
Assume that the area of the occupied space by the subject tree is circular, so the area will be estimated according to the following formula:

Estimated available area for subjected tree $\mathrm{A}=\pi \overline{\mathrm{R}}^{2}$
$A=\pi\left(D x \frac{\left(\sum_{i=1}^{n} \frac{d_{i}}{D t_{i}}\right)}{2 n}\right)^{2}$
This value was compared with the projected area of the crown canopy of the subject tree i.e. (Acr)
Acr $=\pi\left(\frac{\mathrm{cw}}{2}\right)^{2}=\frac{\pi}{4} * \mathrm{Cw}^{2}=$
$0.785 \mathrm{Cw}^{2}$
Where: Dx is the diameter of the subject tree, i.e. the tree, to which the competition factor is to be calculated. $d_{1}, d_{2}, d_{n}$ are the distances between the subject tree and first, second and $n^{t h}$ respectively. $\mathrm{Dt}_{1}, \mathrm{Dt}_{2}, \mathrm{Dt}_{\mathrm{n}}$ are the diameters of the first, second and $n^{\text {th }}$ of trees surrounding subject tree. n is the number of trees used for calculation the competition factor. Cw is the crown diameter or crown width of subjected tree. Acr is the estimation of the projected area of the subject tree.

## Calculation of the Competition Index (CI)

$\mathrm{CI}=\frac{\text { estimated available area }}{\text { actual projected crown area }}=$
$\frac{\mathrm{A}}{\mathrm{Acr}}$.
If the competition index is one or more than one, so there is no evidences of competition, and this value will be taken equal to 1 . And on the other hand, if the competition index is less than 1, this means that the subject tree is under competition stress. This factor will be used to determine the growth potential of the tree as if it has been living in an open area. This can be expressed mathematically using piecewise inequality as follow:


The uses of the theoretical measure of competition index:

1. It could be used for estimating the growth.
2. It can be used to determine, if it is a time of thinning. This can be done by calculating the competition index. If $\mathrm{CI}<1$, then it is necessary to make a plane for thinning for the purpose of improving the growth rate. And on other hand if $\mathrm{CI} \geq 1$, then, it means that the growing space for trees is enough and there is no need to make planes for thinning yet.

It can be used to estimate the growth potential of trees growing in an open area.

Suppose that the diameter of a tree of Quercus infectoria oliv. in 2012 is 22 cm and it increased to 24 in 2020 . And if the local volume equation for this species is $V=0.040653$ $0.007945 \mathrm{D}+0.0005785 \mathrm{D}^{2}$, then the following procedure shows how to use the competition factor to estimate the periodic growth in the following cases:
a. If $\mathrm{CI}=1.24$
b. If $\mathrm{CI}=0.66$

Answer:
Substituting the value of diameter in 2012 and 2020 in the above equation will yield a volume of $0.145857 \mathrm{~m}^{3}$ and $0.183189 \mathrm{~m}^{3}$ respectively.

The periodic growth $=0.183189-0.145857=$ $0.037009 \mathrm{~m}^{3}$
a. Since the Competition Index of the first case is greater than one, therefore the value of the competition Index will be taken as equal to one. See equation 12. And accordingly the potential periodic growth will remain the same as the calculated periodic growth.
b. In the second case, the competition Index is less than one, and according to eq. 12, the potential periodic growth will be calculated as follow:

The potential periodic grow if the tree lives in an open area:
$\frac{0.037339 \mathrm{~m}^{3}}{0.66}=0.056574 \mathrm{~m}^{3}$

## Modelling of Crown Width over DBH

Measuring of crown width is costly, much difficult, and time consuming, and therefore it is preferable to relate it with some other easily measurable variable such as diameter at breast height in a regression model (Cole, et al., 1994).

This developed regression equation, can then be used for estimating of the crown width of a tree with known diameter. For developing such mathematical relationship, a sample of (108) trees were selected from different regions of Duhok Governorate. Both diameter at breast height and the crown width was measured for all trees in the sample.

The crown width was used as dependent variable and different forms of diameter at breast height will be used as independent variable. Out of twenty three regression equations developed, the equation $\mathrm{Cw}=4.73265 * \exp (0.0157654 * \mathrm{D})$, was finally selected because of being most appropriate equation which fits our dataset. The coefficient of determination of the selected equation was 0.6468 .

## Practical Example for Calculating of our Competition Factor

Suppose that we are requested to calculate the competition factor for a tree with dbh of 30 cm and which is situated at distances of $6 \mathrm{~m}, 8 \mathrm{~m}$, 10 m , and 12 m from trees having diameters $36 \mathrm{~cm}, 20 \mathrm{~cm}, 24 \mathrm{~cm}$ and 40 cm respectively, in the following cases:
a. If the mean of the crown diameter for the subject tree is 10 m
b. If the mean of the crown diameter for the subject tree is 9 m
$\overline{\mathrm{R}}=\frac{\mathrm{Dx}\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{D}_{\mathrm{i}}}{\mathrm{Dt} t_{i}}\right)}{2 \mathrm{n}} \rightarrow \overline{\mathrm{R}}=$
$\frac{\mathrm{Dx}\left(\frac{\mathrm{D}_{1}}{\mathrm{D} t_{1}}+\frac{\mathrm{D}_{2} \mathrm{n}}{\mathrm{Dt}}+\frac{\mathrm{D}_{3}}{\mathrm{Dt}}+\cdots+\frac{\mathrm{D}_{\mathrm{t}}}{\mathrm{Dt}}\right)}{2 \mathrm{n}}=\frac{30\left(\frac{6}{36}+\frac{8}{20}+\frac{10}{24}+\frac{12}{40}\right)}{2(4)}=$
4.825 m
$\mathrm{A}=\pi \overline{\mathrm{R}}^{2}=3.14 *(4.825 \mathrm{~m})^{2}=73.1 \mathrm{~m}^{2}$,
the area of the available space
Acr $=0.7854 \mathrm{cw}^{2}=0.7854 *(10 \mathrm{~m})^{2}=$ $78.54 \mathrm{~m}^{2}$, the actual projected area

Since the available space is less than the actual projected area of the subject tree, there is a competition between our subjected and the nearest neighbors. In other words there is overlap between the crowns of subject tree and its neighbors, and hence the growth potential of such tree is less than, the case, the tree was growing in an open area
b) In the second case, the actual available area for the subject tree will remain as the same $\mathrm{A}=\pi \overline{\mathrm{R}}^{2}=3.14 *(4.825 \mathrm{~m})^{2}=73.1 \mathrm{~m}^{2}$
But the crown projected will be changed, as follow:

Acr $=0.7854 \mathrm{cw}^{2}=0.7854(9 \mathrm{~m})^{2}=$ $63.72 \mathrm{~m}^{2}$

Since the actual projected area of the crown is less than what is available, so our subject tree is not under stress of competition.

## CONCLUSIONS

1. The result of this study shows that the breast height diameter of the Calabrian Pine has explained about $64.68 \%$ of the total variation in crown width of the tree
2. There is an exponential relationship between crown width of a tree and diameter at breast height.
3. The squared root of diameter explained about $63.42 \%$ of the total variation in crown width of the tree. For each increase of one cm of the square root of the diameter, there will be 0.83 m in the crown width
4. The theoretical method of calculation can be used to determine the potential growth of a tree, as it was living in an open area instead of living in a stand.
5. About $69.12 \%$ of the total variation of diameter growth of Calabrian pine tree was explained by diameter at breast height for open grown trees, while the corresponding ratio was $66.37 \%$ for trees growing within stands
6. The practical measure of competition factor can be used to decide, if it is time for thinning or not. This can be done by calculating the competition factor. If $\mathbf{C I}<\mathbf{1}$, then it is necessary to make a plane for thinning for the purpose of improving the growth rate. And on other hand if $\mathbf{C I} \geq \mathbf{1}$, then, it means that the growing space for trees is enough and there is no need to make planes for thinning yet.

## REFERENCES

Adlard, P. G. (1974). Development of an empirical competition model for individual trees within a stand. Growth models for tree and stand simulation. Res. Notes, 30, 22-37.

Ashraf, M. I., Meng, F. R., Bourque, C. P. A., \& MacLean, D. A. (2015). A novel modelling approach for predicting forest growth and yield under climate change. PloS one, 10 (7), e0132066.

Avery, T. E., \& Burkhart, H. E. (2015). Forest measurements. Waveland Press.

Bella, I.E., 1971. A new competition model for individual trees. Forest science, 17 (3), pp.364372.

Berger, U., \& Hildenbrandt, H. (2000). A new approach to spatially explicit modelling of forest dynamics: spacing, ageing and neighborhood competition of mangrove trees. Ecological Modelling, 132 (3), 287-302.

Biging, G. S., \& Dobbertin, M. (1995). Evaluation of competition indices in individual tree growth models. Forest science, 41 (2), 360-377

Boivin, F., Paquette, A., Papaik, M.J., Thiffault, N., and Messier, C. 2010. Do position and species identity of neighbours matter in $8-15$-year-old post-harvest mesic stands in the boreal mixed wood? For. Ecol. Manage. 260(7): 1124 1131. doi:10.1016/j.foreco.2010. 06.037. Bräker, O.U

Brown, G. S. (1965). Point density in stems per acre. Forest Research Institute, New Zealand Forest Service.

Canham, C.D., Papaik, M.J., Uriarte, M., McWilliams, W.H., Jenkins, J.C., and Twery, M.J. 2006. Neighborhood analyses of canopy tree competition along environmental gradients in new England forests. Ecol. Appl. 16(2): 540 -554. doi:10.1890/10510761(2006)016[0540:NAOCTC]2.0.CO;2. PMID:16711043.

Coates, K.D., Canham, C.D., and LePage, P.T. 2009. Above- versus below-ground competitive effects and responses of a guild of temperate tree species. J. Ecol. 97(1): $118-130$. doi:10.1111/ j.1365-2745.2008.01458.x

Cole, W. G., \& Lorimer, C. G. (1994). Predicting tree growth from crown variables in managed northern hardwood stands. Forest Ecology and Management, 67 (1-3), 159-175.

Contreras, M. A., Affleck, D., \& Chung, W. (2011). Evaluating tree competition indices as predictors of basal area increment in western Montana forests. Forest Ecology and Management, 262 (11), 1939-1949.
Ek, A. R., \& Monserud, R. A. (1974). FOREST: A computer model for simulating the growth and reproduction of mixed species stands. Res. Rep, 2635.

Enquist, B.J., West, G.B., Charnov, E.L., and Brown, J.H. 1999. Allometric scaling of production
and life-history variation in vascular plants. Nature, 401 (6756): 907-911. doi:10.1038/44819.

Furnival, G. M. (1961). An index for comparing equations used in constructing volume tables. Forest Science, 7 (4), 337-341.

Gerrard, D. J. (1969). Competition quotient: a new measure of the competition affecting individual forest trees (Vol. 20). Agricultural Experiment Station, Michigan State University.

Hamilton, G. J. (1969). The dependence of volume increment of individual trees on dominance, crown dimensions, and competition. Forestry: An International Journal of Forest Research, 42(2), 133-144.

Harrington, C. A., \& Reukema, D. L. (1983). Initial shock and long-term stand development following thinning in a Douglas-fir plantation. Forest Science, 29 (1), 33-46.

Hegyi, F. (1974). A simulation model for managing jack-pinstandssimulation. RoyalColl. For, Res. Notes, 30, 74-90.
Husch, B., Beers, T. W., \& Kershaw Jr, J. A. (2002). Forest mensuration. John Wiley \& Sons.

Kaitaniemi, P., \& Lintunen, A. (2010). Neighbor identity and competition influence tree growth in Scots pine, Siberian larch, and silver birch. Annals of Forest Science, 67 (6), 604604.

Kim, M., Lee, W. K., Kim, Y. S., Lim, C. H., Song, C., Park, T., ... \& Son, Y. M. (2016). Impact of thinning intensity on the diameter and height growth of Larix kaempferi stands in central Korea. Forest science and technology, 12(2), 77-87.
Lorimer, C. G. (1983). Tests of age-independent competition indices for individual trees in natural hardwood stands. Forest Ecology and Management, 6 (4), 343-360.

Maleki et al 2015), tested both spatially and non spatially sets of competition indices to quantify the effects of neighboring trees on diameter growth of silver birch trees.
Miller, G. W. (1997). Effect of crown growing space and age on the growth of northern red oak. In: Spiecker, H.; Rogers, R.; Somogyi, Z., comps. IUFRO Proceedings: advances in research in intermediate oak stands; 1997 July 27-30;

Freiburg, Germany. Freiburg, Germany: University of Freiburg: 140-159.

Muller- Landau, H. C., Condit, R. S., Chave, J., Thomas, S. C., Bohlman, S. A., Bunyavejchewin, S., ... \& Harms, K. E. (2006). Testing metabolic ecology theory for allometric scaling of tree size, growth and mortality in tropical forests. Ecology letters, 9 (5), 575-588.

Nebeker T.E. - Hodges J.D. Karr B.K. and Moehring D.M 1985 Thinning Practices in Southern Pines - With Pest Management Recommendations

Neter, J., Kutner, M. H., Nachtsheim, C. J., \& Wasserman, W. (1996). Applied linear statistical models (Vol. 4, p. 318). Chicago: Irwin.

Newnham, R. M. (1964). The development of a stand model for Douglasfir [Ph. D. thesis]. Vancouver: The University of British Columbia.
Ohtomo, E. 1956. A study on preparation of volume table. Jour.Jap.Soc. vol. 36 (5)

Opie, J. E. (1968). Predictability of individual tree growth using various definitions of competing basal area. Forest Science, 14 (3), 314-323.

Pretzsch, H. (2009). Forest dynamics, growth, and yield. In Forest dynamics, growth and yield (pp.1-39).Springer, Berlin, Heidelberg.

Pukkala, T., \& Kolström, T. (1987). Competition indices and the prediction of radial growth in Scots pine.

Rivas, J. C., González, J. Á., Aguirre, O., \& Hernandez, F. J. (2005). The effect of competition on individual tree basal area growth in mature stands of Pinus cooperi Blanco in Durango (Mexico). European Journal of Forest Research, 124 (2), 133-142.

Salih, T. K., Younis, M. S., \& Wali, S. T. (2019). Dendroclimatological Analysis of Pinus brutia Ten. Grown in Swaratoka, Kurdistan RegionIraq. In Recent Researches in Earth and Environmental Sciences (pp. 9-19). Springer, Cham

Schröder, J., \& Gadow, K. V. (1999). Testing a new competition index for Maritime pine in northwestern Spain. Canadian Journal of Forest Research, 29(2), 280-283.

Schröder, J., Soalleiro, R. R., \& Alonso, G. V. (2002). An age-independent basal area increment model for maritime pine trees in northwestern Spain. Forest Ecology and Management, 157 (1-3), 55-64.

Soucy, M., Lussier, J. M., \& Lavoie, L. (2012). Longterm effects of thinning on growth and yield of an upland black spruce stand. Canadian journal of forest research, 42 (9), 1669-1677.

Stadt, K.J., Huston, C., Coates, K.D., Feng, Z., Dale, M.R.T., and Lieffers, V.J. 2007. Evaluation of competition and light estimation indices for predicting diameter growth in mature boreal mixed forests. Ann. For. Sci. 64 (5): 477- 490. doi:10.1051/forest: 2007025

Sumida, A., Miyaura, T., \& Torii, H. (2013). Relationships of tree height and diameter at breast height revisited: analyses of stem growth using 20 -year data of an even-aged Chamaecyparis obtusa stand. Tree physiology, 33(1), 106-118.

Uriarte, M., Canham, C. D., Thompson, J., \& Zimmerman, J. K. (2004). A neighborhood analysis of tree growth and survival in a hurricane- driven tropical forest. Ecological Monographs, 74 (4), 591-614.

Vanclay, J. K. (1994). Modelling forest growth and yield: applications to mixed tropical forests. School of Environmental Science and Management Papers, 537.
von Oheimb, G., Lang, A. C., Bruelheide, H., Forrester, D. I., Wäsche, I., Yu, M., \& Härdtle, W. (2011). Individual-tree radial growth in a subtropical broad-leaved forest: the role of local neighbourhood competition. Forest Ecology and Management, 261 (3), 499-507.

Zhang, S., Burkhart, H. E., \& Amateis, R. L. (1997). The influence of thinning on tree height and diameter relationships in loblolly pine plantations. Southern journal of applied forestry, 21 (4), 199-205.

