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GLOBAL CONVERGENCE OF A MODIFIED DIA-YUAN CONJUGATE GRADIENT FOR UNCONSTRAINED OPTIMIZATION

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ABSTRACT

A new modified formula of the parameter $ß_n$ for the conjugate gradient method CG is proposed in this paper. The CG algorithm is a very effective method for solving unconstrained optimization, especially for large-scale problems. The suggested method based on famous Dia-Yuan method with approaches new γ_n which can give the new algorithm. Generally, the numerator of the $D-Y$ plays a vital role in having strong convergent property. However, it has jamming issue in practice but the new CG algorithm has better numerical results in practice and demonstrates performance well. The new method has descent condition under curvature condition and sufficient descent condition. Furthemore, the global convergence of the proposed method is established as well. That's why, preliminary numerical result compared with standard Dia-Yuan of CG method show that our method is more robust and effective.

***KEYWORDS**:* unconstrained optimization, conjugate gradient method, descent condition, sufficient descent condition,curvature condition and global convergence.

INTRODUCTION

In this paper, we deal with conjugate gradient method that has a special role in solving the non-linear optimization problems. It is an efficient and an organized tool for solving the large-scale optimization problem due to it is easiness,low memory requirement and simplicity [1,2]. Also it don't need to storage matrices. They remain very common method for engineers and mathematician.[17]

In unconstrained optimization problems we hope to minimize the cost function that depend on real variable with no restriction at all on the value of these real variables.

The mathematics form is min $f(x)$ where $f: R^I \to R$
where the function f is continuous and differentiable.

Convex function: A function $f : R^I \to R$ is convex function if Domain f is convex set, for all $x_1, x_2 \in dom(f)$ and θ with $0 \le \theta \le 1$, we have $f(\theta x_1 + (1-\theta)x_2)$.

$\boldsymbol{Notation}$: for a smooth function f at x_n , where its gradient is available. We denote $\varrho(x_n) = \nabla f(x_n)$.

and the iterative formula usually generates a sequence $\{x_n\}$ as

$$x_{n+1} = x_n + \nu_n \qquad (1)$$

where $\nu_n = \varphi_n \rho_n$ (2)--------

and φ_n is a step size that is determined by some line searches (Wolf condition, Goldstein condition, curvature condition and sufficient decrease condition), [10] ρ_n is the search direction defined as

$$\rho_{n+1} = -\varrho_{n+1} + ß_n \rho_n \quad (3)$$

$$\rho_0 = -\varrho_0 \quad (4)$$

Where $ß_n$ is an conjugacy parameter.
there are well-known formula of $ß_n$ which are defined as

$$ß_n{}^{F-R} = \frac{\varrho_{n+1}^t \varrho_{n+1}}{\varrho_n{}^t \varrho_n} \quad (5)$$
$(Fletcher - Reeves(1964)[7])$

$$ß_n{}^{D-Y} = \frac{\varrho_{n+1}^t \varrho_{n+1}}{\rho_n{}^t \gamma_n} \quad (6)$$
$(Dia - Yuan(1999)[5])$

$$ß_n{}^{C-D} = \frac{-\varrho_{n+1}^t \varrho_{n+1}}{\rho_n{}^t \varrho_n} \quad (7)$$
$(Conjugate - Descent(1997)[8])$

The above methods have strong convergent properties, but they may not have performance well in practice because of the jamming problems [11]. Moreover, the other three common methods are stated as

$$ß_n{}^{PR} = \frac{\varrho_{n+1}^t \gamma_n}{\varrho_n{}^t \varrho_n} \quad (8)$$
$(polak - Riber(1969)[14])$

$$ß_n{}^{H-S} = \frac{\varrho_{n+1}^t \gamma_n}{\rho_n{}^t \gamma_n} \quad (9)$$
$(Hestenes - Stiefel(1952)[9])$

$$ß_n{}^{L-S} = \frac{-\varrho_{n+1}^t \gamma_n}{\rho_n{}^t \varrho_n} \quad (10)$$
$(Lia - Storey(1991)[12])$

Generally the above state methods are not convergent , but they often perform better. Naturally, scientists try to suggest some new methods, which have the advantages of this two kinds of method. For instance, recently (Andrei) [11] suggested a new hybrid conjugate gradient method based on combining both $(H - S)$ and $(D - Y)$ methods for large-scale optimization problem.

consider the Euclidean norm $\|.\|$ and

$$\gamma_n = \varrho_{n+1} - \varrho_n \quad (11)$$

Many numerical methods for unconstrained optimization are proved to converge under Wolf condition [15]

$$f(x_n+\varphi_n\rho_n)-f(x_n)\le \mu\varphi_n{\varrho_n}^t\rho_n \quad (12)$$

$$\varrho(x_n+\varphi_n\rho_n)^t\rho_n\ge\sigma{\varrho_n}^t\rho_n \quad (13)$$

Definition: [11,17] In one hand, for conjugate gradient method in iterative formula in (1) and search direction in (3), we say that the descent condition holds if

$$\varrho_{n+1}^t\rho_{n+1}\le 0, \quad \forall\ n\ge 1 \quad (14)$$

On the other hand, we say that the sufficient condition holds,[18] if there exist $a>0$ such that

$$\varrho_{n+1}\rho_{n+1}^t\le -a\parallel\varrho_{n+1}\parallel^2, \quad \forall\ n\ge 1 \quad (15)$$

The rest of the paper is organized as follows: **in the next section,** the motivation and formula for a contraction of new conjugate gradient method with its algorithm is given. The descent condition and sufficient condition of the new algorithm are proved under suitable condition presented in **section 3**. But **section 4** illustrates some assumptions and analyze its global convergence. **In section 5,** we discuss some interesting numerical results and comparing new numerical result with other CG method which is also established. Moreover, we make summery for our paper that shows in **section 6** .

2motivation And A New Formula $\{ß_n^{new(D-Y)^\star}\}$

2.1 Derivation of $ß_n^{new(D-Y)^\star}$

In this section we derive a new conjugate gradient parameter for unconstrained optimization and relates to the $\beta(D-Y)$ by using some new approaches (δ and γ_n) for finding the minimum of the continuous function.

Consider

$$\gamma'_n=(1-\theta)\delta\,\gamma_n \quad (16)$$

$$\delta=\frac{{\gamma_n}^t\gamma_n}{{\nu_n}^t\gamma_n} \quad (17)$$

where $\theta\in(0,1)$, and the standard parameter is shows in equation(6) .

Put the equations(16 ,17) to equation (6), we get

$$ß_n=\frac{\varrho_{n+1}^t\varrho_{n+1}}{{\rho_n}^t\gamma_n(1-\theta)\delta} \quad (18)$$

$$ß_n=\frac{\varrho_{n+1}^t\varrho_{n+1}}{{\rho_n}^t\gamma_n(1-\theta)\frac{{\gamma_n}^t\gamma_n}{{\nu_n}^t\gamma_n}} \quad (19)$$

$$ß_n=\frac{\parallel\varrho_{n+1}\parallel^2\,{\nu_n}^t\gamma_n}{{\rho_n}^t\gamma_n(1-\theta)\parallel\gamma_n\parallel^2} \quad (20)$$

So by equation (2) we can rewrite equation (20) as follows:

$$ß_n^{new(D-Y)^\star}=\frac{\varphi_n\parallel\varrho_{n+1}\parallel^2}{(1-\theta)\parallel\gamma_n\parallel^2} \quad (21)$$

2.2 Algorithm of $ß_n^{new(D-Y)^\star}$

step 1: set $n=0$, select initial point $x_n\in R^I$
step 2: $\varrho_n=\nabla f(x_n)$, if $\varrho_n=0$ then stop
Else, set $d_n=-\varrho_n$
step 3: compute step length φ_n to minimize $f(x_{n+1})$
step 4: $x_{n+1}=x_n+\varphi_n\rho_n$
step 5: $\varrho_{n+1}=\nabla f(x_{n+1})$
step 6: compute $ß_n^{new(D-Y)^\star}$ where $ß_n^{new(D-Y)^\star}$ in equation (21)
step 7: $\rho_{n+1}=-\varrho_n+ß_n^{new(D-Y)^\star}\rho_n$
step 8: If $n=I$ or $|{\varrho_{n+1}}^t\varrho_n|\ge 0.2\parallel\varrho_{n+1}\parallel^2$ is satiesfies, then go to step 2
Else $n=n+1$ then go to step 3

3Descent and the Sufficient Descent Conditions of the New Conjugate Gradient method $ß_n^{new(D-Y)^\star}$

Theorem (3.1)

Suppose that the sequence $\{x_n\}$ is produced by an equation (1) where the step size defined by wolf line search. Then the search direction ρ_{n+1} with modified parameter β^{D-Y} of conjugate gradient method is a descent direction that shows in equation (14), in both cases exact and inexact line search.

Proof: the search direction with the new conjugate gradient method ($ß_n^{new(D-Y)^\star}$) is

$\rho_{n+1}=-\varrho_{n+1}+ß_n^{new(D-Y)^\star}\rho_n$

Multiply both sides of above equation by ϱ_{n+1}^t, thus it gives

$\rho_{n+1}\varrho_{n+1}^{t} = -\varrho_{n+1}^{t}\varrho_{n+1} + \frac{\varphi_n\|\varrho_{n+1}\|^2}{(1-\theta)\|\gamma_n\|^2}\varrho_{n+1}^{t}\rho_n$ (22)

Then

$\rho_{n+1}\varrho_{n+1}^{t} = -\| \varrho_{n+1} \|^2 + \frac{\varphi_n\|\varrho_{n+1}\|^2}{(1-\theta)\|\gamma_n\|^2}\varrho_{n+1}^{t}\rho_n$ (23) $\rho_{n+1}\varrho_{n+1}^{t} = -\| \varrho_{n+1} \|^2 \left(1 - \frac{\varphi_n}{(1-\theta)\|\gamma_n\|^2}{\rho_n}^t\varrho_{n+1}\right)$ (24)

The step length φ_n chosen by the exact line search that consists of $\varrho_{n+1}\rho_n = 0$, then the proof is complete. Other wise choosing φ_n by inexact line search requires that $\varrho_{n+1}\rho_n \neq 0$.

By curvature condition we have

$\varrho_{n+1}^{t}\rho_n \geq b_1{\varrho_n}^t\rho_n$

Where

$$b_1 \in (0,1)$$

We can rewrite the condition as follow

$$-\varrho_{n+1}^{t}\rho_n \leq b_1 \| {\varrho_n}^2 \|$$

Put in equation (24),it gives

$$\rho_{n+1}\varrho_{n+1}^{t} \leq -\| \varrho_{n+1} \|^2 \left(1 + \frac{\varphi_n b_1 \| \varrho_n \|^2}{(1-\theta) \| y_n \|^2}\right)$$

Since $\theta \in (0,1)$, $(1-\theta) \geq 0$

Now the terms $\| \varrho_n \|^2, \| \gamma_n \|^2, b_1$ are positive and $\varphi_n > 0$

Thus

$\left(1 + \frac{\varphi_n b_1\|\varrho_n\|^2}{(1-\theta)\|\gamma_n\|^2}\right) > 0$

Then

$$\rho_{n+1}\varrho_{n+1}^{t} \leq -\| \varrho_{n+1} \|^2 \left(1 + \frac{\varphi_n b_1 \| \varrho_n \|^2}{(1-\theta) \| \gamma_n \|^2}\right)$$

then it satisfies the decent condition.

$$\Rightarrow \rho_{n+1}\varrho_{n+1}^{t} \leq 0$$

Theorem (3.2)

Assume that the modified conjugate gradient method with search direction that in equation (3) where the step size φ_n satisfies wolf condition, then the following sufficient condition

$$\rho_{n+1}\varrho_{n+1}^{t} \leq -a \| \varrho_{n+1} \|^2 \text{ is hold.}$$

Proof : the search direction with new beta is

$$\rho_{n+1} = -\varrho_{n+1} + ß_n^{new(D-Y)^*}\rho_n \quad (25)$$

multiply both sides of (25) by (ϱ_{n+1}^{t}) and substituting equation(21) in (25)

$$\rho_{n+1}\varrho_{n+1}^{t} = -\varrho_{n+1}^{t}\varrho_{n+1} + \frac{\varphi_n \| \varrho_{n+1} \|^2}{(1-\theta) \| \gamma_n \|^2}\varrho_{n+1}^{t}\rho_n$$

Then

$$\rho_{n+1}\varrho_{n+1}^{t} = -\| \varrho_{n+1} \|^2 + \frac{\varphi_n \| \varrho_{n+1} \|^2}{(1-\theta) \| \gamma_n \|^2}\varrho_{n+1}^{t}\rho_n$$

$\rho_{n+1}\varrho_{n+1}^{t} = -\| \varrho_{n+1} \|^2 \left(1 - \frac{\varphi_n\varrho_{n+1}^{t}\rho_n}{(1-\theta)\|\gamma_n\|^2}\right)$
(26)

By using curvature condition

$$-\varrho_{n+1}^{t}\rho_n \leq b_1 \| \varrho_n \|^2$$

Where $b_1 \in (0,1)$

By using the condition in equation (26), we get

$$\rho_{n+1}\varrho_{n+1}^{t} = -\| \varrho_{n+1} \|^2 \left(1 + \frac{\varphi_n b_1 \| \varrho_n \|^2}{(1-\theta) \| \gamma_n \|^2}\right)$$

since $a = 1 + \frac{\varphi_n b_1\|\varrho_n\|^2}{(1-\theta)\|\gamma_n\|^2}$,

$a > 0$ because of

$\varphi_n, b_1, \| \varrho_n \|^2, \| \gamma_n \|^2$ are positive terms.

Then it satisfies the equation (15)

4 The Global Convergence Condition Analysis of the New Conjugate Gradient method

In this section, the global convergence of a new method is established, so we need to define the following basic assumptions on the objective function [6].

Assumption ($\boldsymbol{E}$)

$(E1)$: the level set $\Omega = \{x \in R^I | f(x) \leq f(x_0) + \epsilon\}$ is bounded

$(E2)$: in a neighborhood η of Ω , f is continuously differentiable and its gradient is Lipschitz continuous, i.e, there exist a constant $L > 0$ s.t

$$\| \varrho(x_{n+1}) - \varrho(x_n) \| \leq L \| x_{n+1} - x_n \|, \quad \forall x_{n+1} - x_n \in \eta$$

Preposition (4.1): Under the assumption (E) of f there exist a constant $\tau \geq 0$ such that

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$$\| \varrho_{n+1} \| \le \tau \qquad (27)$$

Lemma (1): suppose that the assumption (E) is true and suppose any conjugate gradient $(x_{n+1})$and(ρ_{n+1}) , where descent direction ρ_n and φ_n are obtained from the wolf condition if

$$\sum_{n\ge1} \frac{1}{\|\rho_{n+1}\|^2} = \infty \qquad (28)$$

then

$$\liminf_{n\to\infty} \| \varrho_{n+1} \| = 0 \qquad (29)$$

If the function f is uniformly convex function, then there exist a constant $\theta \ge 0$ such that

$$(\varrho(x_{n+1}) - \varrho(x_n))^t (x_{n+1} - x_n) \ge \theta \| x_{n+1} - x_n \|^2$$
(30)

We can rewrite the above equation in another way as it follows

$$\gamma_n{}^t \nu_n \ge \theta \| \nu_n \|^2 \qquad (31)$$

Theorem (4.2): If the assumption (E) is true and that f is uniformly convex function. Then the new method of the form (1),(3) and (21) where φ_n is obtained from wolf line search (13) is satisfies the global convergence $i.e$

$$\liminf_{n\to\infty} \| \varrho_{n+1} \| = 0$$

Proof: from the equation (3) and equation (21) we have

$$\rho_{n+1} = -\varrho_{n+1} + ß_n{}^{new(D-Y)^\star} \rho_n$$

$$ß_n{}^{new(D-Y)^\star} = \frac{\varphi_n \| \varrho_{n+1} \|^2}{(1-\theta) \| \gamma_n \|^2},$$

where $\theta \in (0,1)$

$$|ß_n{}^{new(D-Y)^\star}| = \left| \frac{\varphi_n \| \varrho_{n+1} \|^2}{(1-\theta) \| \gamma_n \|^2} \right|$$

under the assumption (E) and equation (27) we have

$$|ß_n{}^{new(D-Y)^\star}| \le \left| \frac{\varphi_n \tau^2}{(1-\theta) L \| \nu_n \|^2} \right|$$

Since

$$\| \rho_{n+1} \| \le \| \varrho_{n+1} \| + \left| ß_n{}^{new(D-Y)^\star} \right| ||\rho_n||$$

Then

$$\| \rho_{n+1} \| \le \tau + \frac{\varphi_n \tau^2}{(1-\theta) L \| \nu_n \|^2} \| \rho_n \|$$

And by using equation (2), it gives the form

$$\| \rho_{n+1} \| \le \tau + \frac{\tau^2}{(1-\theta) L \| \nu_n \|} \qquad (32)$$ (26)

Since

$$\chi = max \| x_{n+1} - x_n \| $$ (27)

Consider the equation (32) will be of the form (28)

$$\| \rho_{n+1} \| \le \tau \left(1 + \frac{\tau}{L(1-\theta)\chi} \right) \qquad (33)$$

let

$$\mu = \tau\left(1 + \frac{\tau}{L(1-\theta)\chi}\right) \qquad (34)$$

Hence using (34) in the quatuion (33) ,it gives (30)

$$\sum_{n\ge1} \frac{1}{\| \rho_{n+1} \|^2} \ge \sum_{n\ge1} \frac{1}{\mu^2} = \frac{1}{\mu^2} \sum_{n\ge1} 1 = \infty$$

$$\Rightarrow \sum_{n\ge1} \frac{1}{\| \rho_{n+1} \|^2} = \infty$$

By using lemma (1), we get

$$\liminf_{n\to\infty} \| \varrho_{n+1} \| = 0.$$

5 NUMERICAL PERFORMANCE

In this part we compare the numerical results of the proposed new method with standard method Dia-Youn β^{D-Y}, also compare their performance. The comparative tests consist of well known non-linear problems with 13 different functions where $4 \le I \le 5000$. Moreover the code was written in Fortran 95 language and for all cases the stopping condition $\| \varrho_{n+1} \| \le 1x10^{-5}$, and for restarting we use the Powell condition $|\varrho_n{}^t \varrho_{n+1}| \ge 0.2 \| \varrho_{n+1} \|^2$.

In the other hand, the comparative results are illustrated in table (1) which contain number of iteration (NOI) and the number of function (NOF). The experimental result shown in table (1) verifies that the new conjugate gradient method $(ß_n{}^{new(D-Y)^\star} \rho_n)$ is superior to the standard (D-Y) with respect to the number of iteration (NOI) and the number of function (NOF) .

Table (1): comparing the performance of the two algorithms standard Dia-Youn and new formula $ß_n^{new(D-Y)^*}$

Test functions	N	standard form D-Y		New form $ß_n^{new(D-Y)^*}$	
		NO I	NO F	NO I	NO F
Beal	4	11	28	9	22
	100	12	30	9	22
	500	12	30	9	22
	1000	12	30	9	22
	5000	12	30	11	26
Non-Diagonal	4	24	63	23	61
	100	29	82	29	79
	500	29	82	29	79
	1000	.	.	29	79
	5000	30	80	30	81
G-Central	4	18	127	12	70
	100	20	153	17	130
	500	23	192	17	130
	1000	23	192	19	156
	5000	24	205	23	213
Cubic	4	14	39	13	39
	100	15	43	13	39
	500	15	43	13	39
	1000	15	43	13	39
	5000	15	43	13	39
Miele	4	36	115	36	113
	100	45	156	45	154
	500	53	188	51	182
	1000	60	222	57	212
	5000	66	257	63	244
Rosen	4	30	82	27	75
	100	30	82	27	75
	500	30	82	27	75
	1000	30	82	27	75
	5000	30	82	27	75
G-Wolf	4	11	23	11	23
	100	45	91	43	86
	500	48	97	47	95
	1000	52	105	50	101
	5000	159	327	159	327
G-Edger	4	5	14	5	15
	100	5	14	5	15
	500	6	16	5	15
	1000	6	16	5	15
	5000	6	16	5	15
G-Wood	4	28	65	26	60
	100	28	65	27	62
	500	29	68	28	64
	1000	29	68	28	64
	5000	29	68	28	64
OSP	4	8	44	8	44
	100	52	180	52	181
	500	138	439	118	355
	1000	196	607	170	511
	5000	555	1857	398	1278
Sum	4	3	11	3	11
	100	14	85	14	85
	500	21	118	21	116
	1000	24	125	22	104
	5000	33	149	30	152
G-Powell 3	4	14	33	12	29
	100	14	33	12	29
	500	14	33	13	31
	1000	14	33	13	31
	5000	15	35	13	31
Shallow	4	8	21	8	21
	100	8	21	8	21
	500	8	21	8	21
	1000	9	24	8	21
total		2403	7829	2138	6784

Table 2: comparing the rate of improvement between standard method Dia-Youn and new method $ß_n^{new(D-Y)^*}$

Tools	(D-Y)	($ß_n^{new(D-Y)^*}$)
NOI	100%	88.9721%
NOF	100%	86.6521%

Table (2) gives the rate of improvement in the new method $(D-Y)^*$ with the standard method. The numerical performance of the new method is better than the standard algorithm, as we noticed that (NOI) and (NOF) of the standard methods are around 100%. This means that new algorithm has improvement as compared to the standard with(11.0279) in (NOI) and (13.3478) in (NOF).

In general, the new method has been improved by (12.1879) as compare to the standard (D-Y).

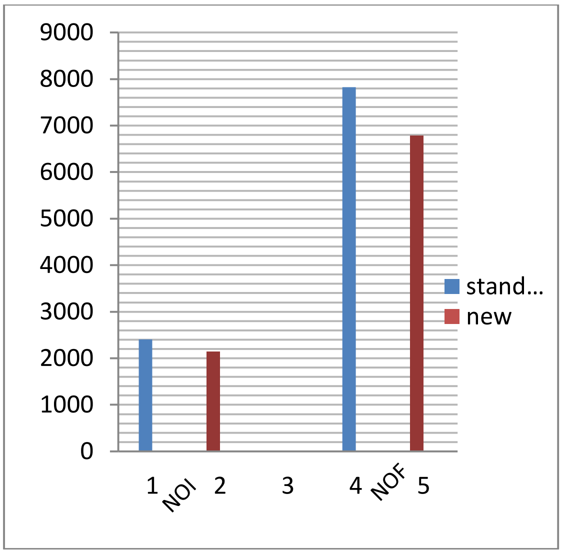

Fig.(1): presents the comparison between new algorithm ($ß_n^{new(D-Y)^*}$) and the standard form of (D-Y) according to the total number of iterations (NOI) and the total number of functions (NOF).

6 CONCLUSION

This paper presented a modified conjugate gradient method for solving a non-linear unconstrained optimization in (21) by changing some approaches. Moreover, in this paper we illustrated that the search direction with the new method satisfies the descent, sufficient condition and global convergence condition. the numerical results shows that the new method has better performance than the standard (D-Y) algorithm by using some standard test functions.

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پوختە

شێوازێ نوی یێ پەیسكێن هاوشێوە مە پێشنییاركرییە دفێ فە كولینێدا. پەیسكێن هاوشێوە شێوازەكێ گەلەك گرنگ بو شیكاركرنا نمونەیێن نەگرێدای ,ئەف شێوازێ پێشنیاركری پشت بەستییە ب شێوازێ (D- Y) دگەل لێكجوییێن (Yn) بومە شێوازێ نوی دەركەت .

هژمارەیێن (D- Y) كو رولەكێ گرنك هەیە دتایبەتمەندییا نێزیكبونا گشتی دا. وشێوازێ نوی

ئەنجامەکێ باشتر هەیە دهژمارەییان دا و ل جێبەجێکرنێ و هەروەسا مەرجێ لاری و لارییا کافی و مەرجێن نێزیك بوناگشتی جێبەجێکرییە. و ئەنجامێن هژمارەیێن دەسپێکی ب بەراورد دگە ل شێوازێ ستاندارد دیارکرییە کو شێوازێ نوی گەلەك کاریگەرتر و ب هێزترە.

الخلاصة

تم أقراح صيغة محسنة جديدة لعامل الترافق $ß_n$ لطريقة المتجهات المترافقة. طريقة المتجهات المترافقة فعالة جدا لحل مسائل الامثلية غير مقيدة و خاصة في مسائل ذات الابعاد الكبيرة. الطريقة المقترحة تعتمد على طريقة (Dia-Youn) المشهورة مع أقتراب جديدة ل(γ_n) .

خوارزميتنا الجديدة كما في طريقة (Dia-Youn) تمتلك خاصية حيوية في التقارب القوي الشامل ولكن في طريقة (D-Y) فيها تشويش في التطبيق و بالتالي خوارزمية CG الجديدة تمتلك نتائج عددية افضل من الطريقة السابقة و اداؤها جيد.

الطريق الجديدة تمتلك خاصية الانحدار بوجود شرط الانحناء و شرط الانحدارالكافي. كذالك تم اثبات شرط التقارب الشامل للطريقة المقترحة.

تم مقارنة النتائج العددية مع طريقة Dia-Youn القياسية و ظهرت ان طريقتنا الجديدة افضل و اكثرفعالية من الطريقة السابقة.