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BEST ROBUST TO ESTIMATE THE PARAMETERS USING HAMPEL WEIGHTED RIDGE LEAST TRIMMED SQUARES IN PRESENCE OF MULTICOLLINEARITY AND OUTLIERS

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ABSTRACT

The production of the undesirable impacts on the least squares estimators is by multicollinearity and outliers which are considered as problems in multi regression models. In the current study, an experimental comparative investigation is made for diverse estimation methods. which namely the Ordinary Least Squares (LS), Ridge Regression (RID), Hampel Weighted Ridge Least Absolute Value (HRLAV) and Hampel Weighted Ridge Least Trimmed Squares (HRLTS). From a numerical example and a simulation study, the resulting Hampel Weighted Ridge Least Trimmed Squares (HRLTS) is efficient than other estimators, using the Standard Error (SE) for real data and Root Mean Squared Error (RMSE) criterion for normal disturbance distribution and different degree of multicollinearity.

KEYWORDS: Multicollinearity; Outliers; Ridge regression; Robust Ridge regression.
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1. INTRODUCTION

In regression analysis, there are two significant issues, namely: multicollinearity and outliers. The regular least square estimators (LS) of coefficients are shown to have some optimal properties when there is no correlation among explanatory variables. The information will not be precise about the regression coefficients when there is correlation among the variables. To add more, there will very poor estimates produced by the least square estimator when outlines exist. Therefore, some diverse remedial techniques have been put forward for each problem in isolation.

Ridge regression is a curative technique when dealing with multicollinearity and the outliers do not powerfully affect the robust estimation techniques. Despite of the fact of thinking of the two problems each in isolation, there is a simultaneous occurrence of both of them. Montgomery and Peck (1982) have stated that one of the methods of ridge estimation or robust estimation can be enough in dealing with combined problem.

For solving the two problems that occur at the same time, many robust ridge regression estimators have been suggested in the sense that multicollinearity and outliers do not affect vastly the estimators. To make a combination of the ridge regression techniques of both the least absolute deviation (LAD) robust and the ridge was the notion proposed by Askin and Montgomery (1980). The current study adopts the idea of combining as the researcher attempts to remedy the two problems by developing a more robust technique.

The Hampel weighted function with robust ridge regression, called the WRLAV and the Hampel weighted function with ridge regression, known as HRLTS, as high breakdown point estimator. Depending on both RLAV and RLTS, the researcher calls the method modified as the robust ridge regression. It is assumed that there will be less sensitivity of the method modified towards the problems of both multicollinearity and outliers.

Therefore, this paper aims at examining some estimators that resist the problems of multicollinearity and outliers combined. To be more precise, the combination of ridge and robust estimators will be more effective than each one of them in isolation. In section (2) and (3), the techniques of ridge estimator and robust estimation will be discussed, respectively while the combination of both will be demonstrated in section (4). Weighted robust ridge regression is discussed in Section 5. Section (6) and Section

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(7) presents the results of a numerical example and Monte Carlo simulation study to investigate how such estimators perform well and some conclusion is presented in Section (8).

2. RIDGE REGRESSION ESTIMATORS

Consider the following linear regression model

$$Y = X\beta + \varepsilon, \qquad (1)$$

where y is an ($n \times 1$) vector of observations on the response variable, X is an ($n \times p$) matrix of observations on the explanatory variables, β is a ($p \times 1$) vector of regression coefficients to be estimated and ε is an ($n \times 1$) vector of disturbances. The least squares estimator of β can be written as:

$$\hat{\beta}_{LS} = \left(X'X\right)^{-1} X'Y \qquad (2)$$

Among all the neutral linear estimators, this method provides variance that is minimum and unbiased. To add more, there is independent, normal, and identical distribution of the errors. In contrary, when there is multicollinearity, the singularities are $\left(X'X\right)$ matrix and this ill-conditioned X matrix can result in very poor estimates.

There will be improvement of the conditioning of a matrix and this will lead to the reduction of VIF when there is addition of a small constant to the diagonal of a matrix. The definition of the ridge regression is as follows (Hoerl and Kennard, 1970):

$$\hat{\beta}_{RID} = \left(X'X + kI\right)^{-1} X'Y$$

(3)

Where I is the ($p \times p$) identity matrix and k is the biasing constant. In the literature, there have been diverse methods that determine k value, as provided by Hoerl and Kennard (1970) and (Gibbons, 1981) as:

$$k = \frac{pS_{LS}^2}{\hat{\beta}'_{LS}\hat{\beta}_{LS}} \qquad (4)$$

where,

$$S_{LS}^2 = \frac{\left(Y - X\hat{\beta}_{LS}\right)'\left(Y - X\hat{\beta}_{LS}\right)}{n-p}$$

(5)

when k= 0, $\hat{\beta}_{RID} = \hat{\beta}_{LS}$, when k> 0, $\hat{\beta}_{RID}$ is biased but more stable and precise, than LS estimator but when $k \to \infty$, $\hat{\beta}_{RID} \to$ 0. (Hoerl and Kennard, 1970) have shown that, there always exist a value k> 0such that MSE ($\hat{\beta}_{RID}$) <MSE ($\hat{\beta}_{LS}$).

3. Robust Regression Estimators

There is more reliability and efficiency of robust regression estimators than least square estimators. This is the case in the situation that there are many heavy and fatter tails of the disturbances compared with the normal distribution and there is tendency towards the production of outliers. There is much influence of outliers on estimated coefficients, statistics and standard errors; because of the precise impact on the estimator, the statistical procedure is the least efficient. Several different classifications of robust regression exist. Two of the most commonly considered groups are LAV and LTS estimators. The first estimator is LAV can be defined as the solution to the following minimization problem

$$\min \sum_{i=1}^{n} \left| Yi - X_i'\beta_{LAV} \right|$$

(6)

Rather than minimizing the sum of squared residuals as in least squares estimation, the sum of the absolute values of the residuals is minimized. Thus, the effect of outliers on the LAV estimates will be less than that on LS estimates.

The second estimator is LTS this estimator was introduced by (Rousseeuw and Leroy, 1987) as

$$\hat{\beta}_{LTS} = \min \sum_{i=1}^{h} \varepsilon_i^2$$

(7)

The sum of the square residuals is minimized rather than minimizing the sum of squared residuals as in least squares estimation. Thus, this estimator will be effect of outliers less than that on LS estimates.

4. ROBUST RIDGE REGRESSION ESTIMATORS

The idea of making a combination of robust the techniques of biased and robust regression has been explained by Askin and Montgomery (1980). The employment of the procedure of weighted least squares and the computation of the robust estimates are counted as facts on which the combination procedure is based on. In dealing with each problem at the same time, the combined method is the best solution in the case of the occurrence of both multicollinearity and outliers in a data set. The presentation of some of ridge and robust regression estimation is necessary here, but a full account of them will be demonstrated in sections (2) and (3). (Pfaffenberger and Dielman 1984 and 1985) proposed combining the ridge and the Least Absolute Value (LAV) robust regression techniques. In this paper, we take the initiative to develop a more robust technique to rectify these two problems. We proposed combining the ridge regression with the high breakdown point estimator, namely the RLTS. The estimator RLAV can be written as:

$$\hat{\beta}_{RLAV} = \left(X'X + k_{LAV} I\right)^{-1} X'Y \quad , \tag{8}$$

Where the value of k_{LAV} is determined from data using:

$$k_{LAV} = \frac{pS^2_{LAV}}{\hat{\beta}'_{LAV}\hat{\beta}_{LAV}} \tag{9}$$

and

$$S^2_{LAV} = \frac{\left(Y - X\hat{\beta}_{LAV}\right)'\left(Y - X\hat{\beta}_{LAV}\right)}{n-p} \quad , \tag{10}$$

$\hat{\beta}_{RLAV}$ is the LAV estimator defined as the solution to equation (6). It be noted that the value of k_{LAV} is the estimator of *k* presented by equation (4) with two changes. First, the LAV estimator of β is used rather than LS estimator. Second, the estimator of S^2_{LAV} used in equation (10) is modified by the LAV coefficient estimates rather than the least squares estimates. In addition, another robust ridge estimator is RLTS estimator can be written as:

$$\hat{\beta}_{RLTS} = \left(X'X + k_{LTS} I\right)^{-1} X'Y \tag{11}$$

where the value of k_{LTS} is determined from data using:

$$k_{LTS} = \frac{pS^2_{LTS}}{\hat{\beta}'_{LTS}\hat{\beta}_{LTS}} \tag{12}$$

and

$$S^2_{LTS} = \frac{\left(Y - X\hat{\beta}_{LTS}\right)'\left(Y - X\hat{\beta}_{LTS}\right)}{n-p} \tag{13}$$

$\hat{\beta}_{RLTS}$ is the LTS estimator defined as the solution to equation (7). It be noted that the value of k_{LTS} is the estimator of *k* presented by equation (4) with two changes. First, the LTS estimator of β is used rather than LS estimator. Second, the estimator of S^2_{LTS} used in equation (13) is modified by the LTS coefficient estimates rather than the least squares estimates. These changes are aimed to reduce the effect of extreme points on the value chosen for the biasing parameter.

5. WEIGHTED ROBUST RIDGE REGRESSION ESTIMATORS

We suggested using Hampel weighted function to compute the WRLAV and WRLTS estimates; (Andrews et al. 1972) was proposed by Hampel in the “Princeton Robustness Study” and is defined as follows

$$W(H) = \begin{cases} 1 & |x| \le a \\ \dfrac{a}{|x|} & a < |x| \le b \\ \dfrac{a}{|x|}\dfrac{c-|x|}{c-b} & b < |x| \le c \\ 0 & O.W. \end{cases} \tag{14}$$

where the constant a, b, c are called tuning constants satisfying $0 < a \le b < c < \infty$.

Finally, in this respect, the proposed Hampel weighted robust ridge estimator, called Hampel weighted ridge least absolute values estimator can be used to determine the biasing parameter *k* as

$$k_{WLAV} = \frac{PS^2_{WLAV}}{\hat{\boldsymbol{\beta}}'_{WLAV}\hat{\boldsymbol{\beta}}_{WLAV}} \tag{15}$$

where the WLAV estimation procedure of β is used rather than LS estimator in computing the k and S^2 values in order to reduce the effect of no normality on the value, chosen the S^2 value as

$$S^2_{WLAV} = \frac{\left(Y - X\hat{\boldsymbol{\beta}}_{WLAV}\right)'\left(Y - X\hat{\boldsymbol{\beta}}_{WLAV}\right)}{n-p} \tag{16}$$

In this case, the ridge WLAV estimator of the parameter β is given by

$$\hat{\beta}_{WRLAV} = \left(X'WX + k_{WLAV}I\right)^{-1}X'WY \ , \tag{17}$$

where, k_{WLAV} are given in equation (15) and the Hampel weights are determined from equation (14).

As for the Hampel weighted ridge least trimmed square estimator we can apply the same procedure for Hampel weighted ridge least absolute values estimator replace the estimator of the LTS instead of LAV estimator.

6. Numerical Example

A Diabetes data set taken from (Bradley et al., 2004) is adopted for the evaluation of the performance of these two proposed estimators. The ten explanatory values (age, average blood, sex, six blood serum measurements, and body mass index) and the response variable *y*. are contained in this data. Each of 442 diabetes patients showed one response variable. The response of interest is a quantitative measure of disease progression after one year.

The degree of multicollinearity is often indicated by Variance Inflation Factor (VIF) given as $VIF = \frac{1}{1-R^2}$, R^2 is the determinant of the matrix $\mathbf{X'X}$. If VIF>10 indicating the existence of multicollinearity in the data. Table 1 shows VIF for the explanatory variables of the real data especially the VIF for some variables high the variables are X4, X5, X6 and X8 > 10.

Likewise, we can identify the outliers in the data by computing the residuals associated with LMS regression.

$$s = 1.4826\left(1 + \frac{5}{n-p}\right)\sqrt{med\left(\varepsilon_i^2\right)} \ ,$$

where i=1, 2… n, and *med* is the median of the squared residuals, p is the number of explanatory variables. The points $\left(y_i, x_{i1}, ..., x_{ip}\right)$ are labeled as regression outliers if the corresponding standardized residual is large. In particular (Rousseeuw and Van Zomeren, 1990) labeled the *i-th* vector a regression outlier if $|\varepsilon_i|/s > 2.5$, implies the value is outlier. The ordinary or simple residuals (observed- predicted values) are the most commonly used measures for detecting outliers.

The data contained 25 outliers while these outliers affected the data and give the poor estimates.

Table (1): The VIF for the diabetes data

Variable	VIF
X1	1.311
X2	1.723
X3	1.489
X4	**59.677**
X5	**39.303**
X6	**15.389**
X7	8.914
X8	**10.482**
X9	1.462
X10	2.073

Table (2): Estimated parameters $\hat{\beta}_1$, $\hat{\beta}_2$, ..., $\hat{\beta}_{10}$ and SE of diabetes for the proposed methods (HRLAV and HRLTS) and existing methods (OLS, RID) using Hampel weighted function.

Coef.	Estimate	OLS	RID	HRLAV	HRLTS
$\hat{\beta}_1$	parameter	2.620	0.096	0.070	0.093
	S.E.	1.312	0.043	0.085	0.042
$\hat{\beta}_2$	parameter	-0.009	0.006	0.009	0.014
	S.E.	0.170	0.048	0.091	0.046
$\hat{\beta}_3$	parameter	0.199	0.194	0.192	0.191
	S.E.	0.051	0.045	0.088	0.044
$\hat{\beta}_4$	parameter	-0.051	0.069	0.071	0.076
	S.E.	0.128	0.036	0.058	0.032
$\hat{\beta}_5$	parameter	0.103	0.063	0.038	0.060
	S.E.	0.118	0.043	0.072	0.040
$\hat{\beta}_6$	parameter	0.135	0.054	0.064	0.054
	S.E.	0.174	0.048	0.083	0.045
$\hat{\beta}_7$	parameter	-0.388	-0.031	-0.009	-0.033
	S.E.	1.324	0.059	0.092	0.053
$\hat{\beta}_8$	parameter	4.567	0.091	0.095	0.089
	S.E.	3.547	0.049	0.090	0.047
$\hat{\beta}_9$	parameter	0.166	0.135	0.136	0.125
	S.E.	0.060	0.046	0.089	0.044
$\hat{\beta}_{10}$	parameter	-0.002	0.001	-0.006	0.006
	S.E.	0.011	0.050	0.094	0.048

We compared the results of the SE from the Table 2 for all methods see that the SE for the proposed method HRLTS less than the other proposed method HRLAV and the existing methods RID and OLS for all parameters.

7. SIMULATION STUDY

In comparing the performance of some selected alternative estimators combined, this research adopts Monte Carlo simulation. It allows the simultaneous occurrence of both multicolinearity and outliers, and the diverse degrees of multicolinearity. Besides, for the generation of the outliers, the regular disturbance distribution is employed.

The study contains four estimators which are
(1) The least squares estimator (LS).
(2) The ridge regression estimator (RID).
(3) The Hampel weighted ridge least absolute value estimator (HRLAV).
(4) The Hampel weighted ridge least trimmed square estimator (HRLTS).

Suppose, we have the following linear regression model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$$

where i=1,2,…,n (18)

The parameters values $\beta_0, \beta_1, \beta_2$ *and* β_3 are set equal to one [11]. The explanatory variables x_{i1}, x_{i2} and x_{i3} are generated as:

$$x_{ij} = (1-\rho^2) z_{ij} + \rho z_{ij}$$ where i=1, 2,…, n, j =1, 2, 3 (19)

The values of ρ^2 between two explanatory variables selected are0.0, 0.5 and 0.99. The generation of the explanatory variables values is made once for the sample size *n*. The significant factor employed in the current study is the regular disturbance distribution and there is examination of 25, 50 and 100 as sample sizes.

In this case, normal disturbance distribution is generated independently of the explanatory variables used 0.20 of outliers. Programs were written in Rx64 3.0.3. Simulation study was carrying out for every 500 Monte Carlo trials.

We compare the results of simulated data for the following tables using the Root Mean Squared Error (RMSE) to evaluate the resistance of these estimators.

Table (3): Values of Bias, RMSE and SE associated for the estimation of parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$ for proposed and existing methods using Hampel weighted function for ρ =0.0 and 0% of outliers

Prob. Distribution of Error	Sample size	Par.	Est.	OLS	RID	HRLAV	HRLTS
Normal	25	$\hat{\beta}_1$	Bias	0.015	-0.493	-0.498	-0.496
			RMSE	0.227	0.507	0.512	0.517
			SE	0.226	0.117	0.120	0.147
		$\hat{\beta}_2$	Bias	-0.001	-0.496	-0.500	-0.499
			RMSE	0.221	0.510	0.515	0.518
			SE	0.221	0.120	0.121	0.139
		$\hat{\beta}_3$	Bias	-0.001	-0.505	-0.509	-0.506
			RMSE	0.229	0.521	0.525	0.526
			SE	0.229	0.128	0.127	0.144
	50	$\hat{\beta}_1$	Bias	0.005	-0.499	-0.499	-0.494
			RMSE	0.147	0.505	0.506	0.501
			SE	0.147	0.081	0.084	0.085
		$\hat{\beta}_2$	Bias	-0.002	-0.501	-0.503	-0.498
			RMSE	0.144	0.508	0.510	0.506
			SE	0.144	0.082	0.084	0.085
		$\hat{\beta}_3$	Bias	-0.002	-0.500	-0.503	-0.503
			RMSE	0.153	0.510	0.510	0.507
			SE	0.153	0.086	0.086	0.084
	100	$\hat{\beta}_1$	Bias	-0.001	-0.499	-0.501	-0.497
			RMSE	0.100	0.503	0.504	0.500
			SE	0.100	0.057	0.058	0.056
		$\hat{\beta}_2$	Bias	0.004	-0.500	-0.500	-0.500
			RMSE	0.109	0.504	0.504	0.503
			SE	0.109	0.058	0.060	0.054
		$\hat{\beta}_3$	Bias	0.004	-0.499	-0.500	-0.503
			RMSE	0.103	0.503	0.507	0.503
			SE	0.103	0.059	0.060	0.059

From the results of Table 3 under the condition of no multicollinearity and no outliers in the data we see that the OLS estimators for $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$ when the normal disturbance distribution produced less RMSE as compared with the other estimators for all sample size. Therefore, as the OLS estimator is clearly superior to all the estimators, followed by RID also produced less RMSE for 25 sample size and HRLTS for the 50, 100 sample size.

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Table (4): Values of Bias, RMSE and SE associated for the estimation of parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$ for proposed and existing methods using Hampel weighted function for ρ =0.5 and 20% of outliers

Prob. Distribution of Error	Sample size	Par.	Est.	OLS	RID	HRLAV	HRLTS
Normal	**25**	$\hat{\beta}_1$	**Bias**	**-1.399**	**-1.007**	**-0.983**	**-0.980**
			RMSE	**15.596**	**1.048**	**0.983**	**0.981**
			SE	**15.533**	**0.291**	**0.030**	**0.036**
		$\hat{\beta}_2$	**Bias**	**1.134**	**-0.961**	**-0.979**	**-0.979**
			RMSE	**16.588**	**1.011**	**0.979**	**0.980**
			SE	**16.549**	**0.313**	**0.032**	**0.036**
		$\hat{\beta}_3$	**Bias**	**0.582**	**-0.973**	**-0.980**	**-0.981**
			RMSE	**16.359**	**1.022**	**0.980**	**0.982**
			SE	**16.348**	**0.312**	**0.033**	**0.036**
	50	$\hat{\beta}_1$	**Bias**	**-0.378**	**-0.989**	**-0.980**	**-0.979**
			RMSE	**11.060**	**1.016**	**0.980**	**0.979**
			SE	**11.054**	**0.232**	**0.020**	**0.019**
		$\hat{\beta}_2$	**Bias**	**0.324**	**-0.974**	**-0.982**	**-0.980**
			RMSE	**10.717**	**0.999**	**0.982**	**0.980**
			SE	**10.712**	**0.224**	**0.020**	**0.019**
		$\hat{\beta}_3$	**Bias**	**-0.202**	**-0.983**	**-0.981**	**-0.979**
			RMSE	**10.450**	**1.008**	**0.981**	**0.980**
			SE	**10.448**	**0.220**	**0.019**	**0.018**
	100	$\hat{\beta}_1$	**Bias**	**-0.372**	**-0.987**	**-0.981**	**-0.981**
			RMSE	**7.552**	**1.002**	**0.981**	**0.980**
			SE	**7.543**	**0.175**	**0.014**	**0.014**
		$\hat{\beta}_2$	**Bias**	**0.126**	**-0.979**	**-0.981**	**-0.980**
			RMSE	**7.540**	**0.994**	**0.981**	**0.980**
			SE	**7.539**	**0.174**	**0.014**	**0.014**
		$\hat{\beta}_3$	**Bias**	**0.151**	**-0.976**	**-0.981**	**-0.980**
			RMSE	**7.647**	**0.991**	**0.981**	**0.980**
			SE	**7.646**	**0.177**	**0.013**	**0.014**

Table (5): Values of Bias, RMSE and SE associated for the estimation of parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$ for proposed and existing methods using Hampel weighted function for ρ =0.99 and 20% of outliers

Prob. Distribution of Error	Sample size	Par.	Est.	OLS	RID	HRLAV	HRLTS
Normal	25	$\hat{\beta}_1$	Bias	-80.660	-2.639	-1.068	-0.944
			RMSE	75.655	16.917	1.521	1.606
			SE	75.349	16.710	1.083	1.300
		$\hat{\beta}_2$	Bias	55.772	0.267	-0.914	-0.940
			RMSE	78.031	17.495	1.453	1.729
			SE	78.055	17.493	1.129	1.452
		$\hat{\beta}_3$	Bias	25.178	-0.559	-0.950	-1.048
			RMSE	79.664	17.821	1.502	1.662
			SE	79.264	17.812	1.163	1.290
	50	$\hat{\beta}_1$	Bias	-12.079	-1.263	-0.966	-0.941
			RMSE	53.670	12.906	1.326	1.246
			SE	53.533	12.844	0.908	0.817
		$\hat{\beta}_2$	Bias	17.499	-0.553	-0.967	-1.062
			RMSE	51.739	12.652	1.359	1.311
			SE	51.444	12.640	0.885	0.847
		$\hat{\beta}_3$	Bias	-5.467	-1.116	-0.998	-0.928
			RMSE	51.956	12.565	1.326	1.230
			SE	51.927	12.515	0.873	0.808
	100	$\hat{\beta}_1$	Bias	-15.684	-1.339	-0.996	-0.947
			RMSE	36.560	9.842	1.213	1.157
			SE	36.222	9.750	0.692	0.666
		$\hat{\beta}_2$	Bias	6.599	-0.914	-0.989	-0.989
			RMSE	36.387	9.822	1.207	1.181
			SE	36.327	9.779	0.692	0.645
		$\hat{\beta}_3$	Bias	9.182	-0.675	-0.947	-0.996
			RMSE	36.090	9.776	1.178	1.157
			SE	36.974	9.753	0.664	0.630

From Tables (4) and (5), we see that the HRLTS estimator marginally is superior to HRLAV when the ρ= 0.5 and ρ= 0.99 and 20% of outliers for 50 and 100 sample size. Otherwise HRLAV is superior.

In comparing the estimators HRLTS to HRLAV, RID, and LS, the properties of the various estimators are not foreshadowed. As a result, the superiority of the HRLTS estimator is shown over HRLAV over a wide range of values of ρ for the given disturbance normal of the regular distribution as the ridge regression.

8. CONCLUSION

In regression analysis there are two common problems which are multicollinearity and outliers. Despite the fact that of dealing with the two problems in isolation, there is a simultaneous occurrence of the two. Two deals with the two problems, a numerical example and Monte Carlo simulation study were devised for the sake of making a comparison of the performance of robust regression estimators.

The results of comparisons indicate estimator is superior to HRLTS estimator for normal disturbances distribution and degree of multicollinearity Tables (2, 3, 4 and 5). Only, this estimator RID is less efficient than the HRLAV estimator when disturbances are normal. In addition, HRLTS outperforms both HRLAV and RID estimators when the degree of multicollinearity is high. Therefore, the HRLTS estimator appears to be a suitable alterative too there estimators when both multicollinearity and normal disturbances distribution are present. Other authors have discussed the weighting forms can potentially be employed for the construction of the estimators.

REFERENCES

Montgomery D.C. and E.A. Peck (1982). " Introduction to Linear Regression Analysis", Wiley, New York.

AskinR.G. and D.C. Montgomery (1980). " Augmented robust estimators", Technometrics, 22, 333-341.

Rousseeuw, P. J. and Van Zomeren, B. C. (1990). " Unmasking multivariate outliers and leverage points", Journal of the American Statistical Association, 85 (411), 633–63.

-HoerlA.E. and R. W. Kennard (1970)." Ridge Regression: Iterative Estimation of the Biasing parameter", Communications in statistics: A Theory Methods, 5, 77-88.

Gibbons D. (1981). "A Simulation study of some ridge estimators", Journal of American Statistical Association, 76, 131-139.

Rousseeuw P. & Leroy A. (1987). " Robust regression and outlier detection", New York: Wiley.

Pfaffenberger R. C. and T. E. Dielman (1984). " A Modified Ridge Regression Estimator Using the Least Absolute Value Criterion in the Multiple Linear Regression Model", Proceedings of the American Institute for Decision Sciences. Toronto. 791-793.

Pfaffenberger, R. C. and T. E. Dielman (1985). " A Comparison of Robust Ridge Estimators", Proceedings of the American Statistical Association Business and Economic Statistics Section. Las Vegas, Nev., 631-635.

Andrews D.F., Bickel P.J., Hampel F.R. Huber P.J., Rogers W.H. and Tukey J.W. (1972). " Robust Estimates of Location: Survey and Advances", Princeton University Press, New Jersey.

Bradley E. Trevor H. Iain J. and Robert T. (2004)." Least Angle Regression" Annals of Statistics, Volume 32, Number 2, 407-499.

Dempster A.P. Schatzoff M., and N. Wermuth. (1997). "A simulation study of alternatives to Ordinary Least Squares" J.A.S.A., 72, pp. 99.