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PROBABILISTIC SETTLEMENT ANALYSIS OF STRIP FOOTING ON SPATIALLY RANDOM SOIL

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ABSTRACT

The prediction of settlement in a traditional design approach usually uses a deterministic value of modulus of elasticity (E), which is estimated as an average value by testing the soil at selected locations. However, these deterministic properties of soil may not represent the actual properties of soil and site condition. Due to numerous sources of uncertainty, the properties of soil mass are spatially varying and anisotropic in the natural field condition. In this study, a random finite element method (RFEM) is used to evaluate the reliability of settlement of the strip footing on spatially random soil. Modulus of elasticity is the only considered random parameter. For this purpose, 2000 spatially random realizations of E-field are generated using Monte Carlo Simulation. Each of these realizations of heterogeneous soil profile is passed to FEM to analyze the settlement of footing. The final settlement results measured in all these realizations are then statistically evaluated and compared. The results of analysis show that the mean and standard deviation of the footing settlement are increased with increasing spatial correlation length. The large value of isotropic correlation length led to an increase in the mean settlement value by more than 25% as compared with the deterministic settlement calculations. Also, it is concluded that the rate of increase of settlements for anisotropic correlation length is lower than the one under isotropic condition.

KEYWORDS: Foundation settlement; Spatially random soil; Probabilistic analysis; Random Finite element method

1. INTRODUCTION

The settlement prediction and bearing capacity of shallow foundations are usually estimated by using conventional approaches considering the soil as a homogenous porous medium. However, in actual, the soil properties are spatial variables due to some reasons like deposition process, glacial actions, mineralogical composition, and stress history. This random nature of the input data plays a significant role in the reliability of the predicted settlements of the foundations. Therefore, to address this problem probabilistic analysis has been introduced to understand the sensitivity of the result to the variability of the input soil parameters. This approach was developed after the mid-twentieth by Wu & Kraft (1967) and Resendiz & Herrera (1969). In their studies, the soil was modeled as a random field by ignoring the correlation length between random values. Beacher & Ingra (1981) developed and used the stochastic FEM to predict the total and differential settlements of a large flexible footing. Two types of spatial autocorrelation (single exponential and squared exponential) used in their study to link Young's modulus (as random variables) of soil among elements. The study area about probabilistic analysis of settlement of shallow foundations has widely progressed since 1990. A stochastic integral formulation technique was utilized by Zeitoun and Baker (1992) to estimate the settlement of shallow foundations. Paice et al. (1994) evaluated the effect of the random soil stiffness on the settlement prediction of the shallow footing by using a random finite element model.

During the last two decades, this area of study has been focused by several groups of researchers as an important field of study. They have tended to innovate and develop the new methods or techniques of using soil as a spatially random and heterogeneous-medium model to conduct its different settlement, bearing capacity, and seepage analyses. The prediction of total settlement under the spread single footing and differential settlement under a pair of isolated footings are probabilistically investigated by Fenton & Griffiths (2002); Chenari et al. (2019). The stiffness random field was combined with a two dimensional FEM

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through Monte Carlo simulations. In their study, the sensitivity analysis and the effect of mean, variance, and spatial correlation distance of input data on the settlement prediction are investigated. Al-Bittar & Soubra (2014) investigated the vertical and horizontal displacement of strip footing which subjected to a gravity and inclined loads resting on spatially random soil (with considering isotropic and anisotropic random field). The soil mass was simulated by Kenarsari & Chenari (2015) as an anisotropic random field. The generated random fields were linked to the FLAC2D finite-difference model to study the settlement of shallow foundations under this induced soil spatial variability. Recently, the Monte Carlo stochastic finite element program for two-dimensional reliability of foundation settlement has been conducted by Huang et al. (2018).

In this study, the settlement of the strip footing resting on a spatially random soil is probabilistically predicted and analyzed. The influence of the randomly generated stiffness (Young's modulus, E) fields on the settlement prediction is statistically investigated for both isotropic and anisotropic random soils. The Random Finite Element Method (RFEM) is used to model the soil-footing problem. In addition, the accuracy of the settlement prediction and the influence of the spatial fluctuation lengths on the results are studied.

2. RANDOM FIELD MODELING

The assumed soil-foundation system in this study is under a plane strain condition as shown in Fig. (**1**.) The footing is a rigid strip footing of width (w_f) of 1.0m with an applied line load of 100 kN/m. The sufficient soil profile depth is considered of $H = 2.0\,m$ and the vertical boundaries of the model are assumed to be fixed in a horizontal direction (the lateral boundaries). The soil layer is underlain by a rigid stratum at the bottom boundary of the model. The two main parameters of interest in the elastic settlement calculations are Young's modulus E and Poisson's ratio v. Within the concepts of the RFEM, Young's modulus of soil is distributed as a lognormal random variable with mean $\mu_{ln\,E}$, standard deviation $\sigma_{ln\,E}$, isotropic $\theta_{ln\,E\,(isotropic)}$, and anisotropic $\theta_{ln\,E\,(anisotropic)}$ correlation length. Due to the small relative spatial variability of the Poisson's ratio (Fenton & Griffiths 2002; Jimenez & Sitar 2009; Paice et al. 1994), it is held fixed throughout the study at $v = 0.3$.

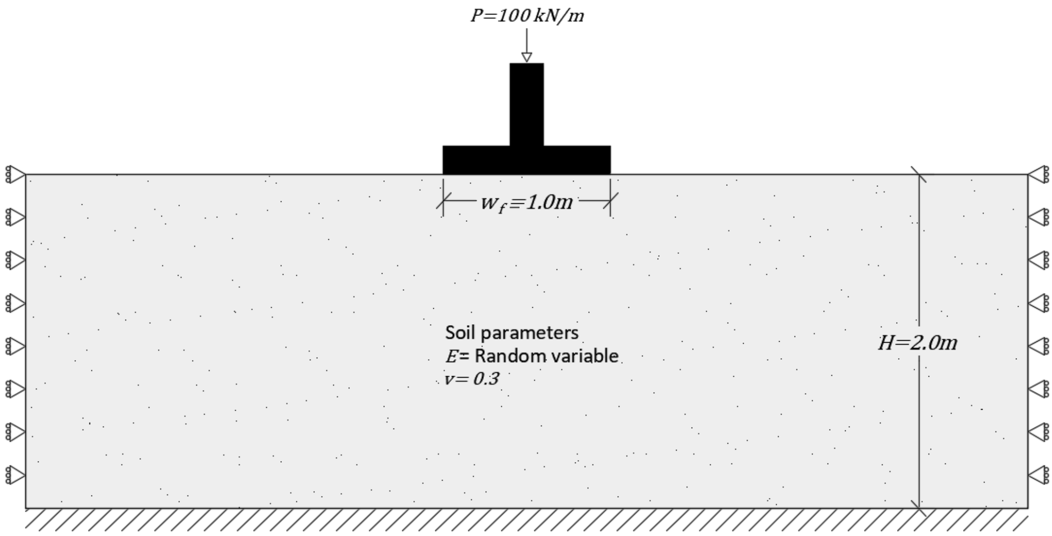

Fig. (1): Geometry of the problem.

The most important numerical characteristics for the random field are mean μ, variance σ^2, and correlation length θ. The considered mean of stiffness (μ_E) throughout this study is 4 MPa. The random E-field is outlined with the coefficient of variation COV_E in the range of 0.1 to 0.5. The correlation distance (scale of fluctuation), describes the spatial variability of soil parameters in the random fields; and it is a distance within which points are significantly related to each other (Vanmarcke, 2010). A small scale of fluctuation indicates that the soil property fluctuates about the mean value. Whereas, a large scale of fluctuation specifies that the soil property is significantly related to a large spatial range. Thus, it is vital to select an

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accurate and suitable correlation distance because it plays a main role in describing the spatial variability of soil properties.

To select a reasonable spatial correlation length, it needs a large amount of data from the site (real-field data) and this cannot be gathered easily especially when the sample size is small. To this end, the autocorrelation function is utilized to describe the spatial correlation of the soil properties. There are several common two-dimensional autocorrelation functions including single exponential, squared exponential, cosine exponential, and second-order Markov as their functions are illustrated in Table (1).

Table (1): Common 2-D autocorrelation function types.

Type	2D Autocorrelation Function
Single exponential	$\rho[\tau_x,\tau_y]=\exp\left[-2(\frac{\tau_x}{\delta_x}+\frac{\tau_y}{\delta_y})\right]$
Squared exponential	$\rho[\tau_x,\tau_y]=\exp\left[-\pi\left(\frac{\tau^2{}_x}{\delta^2{}_x}+\frac{\tau^2{}_y}{\delta^2{}_y}\right)\right]$
Cosine exponential	$\rho[\tau_x,\tau_y]=\exp\left[-(\frac{\tau_x}{\delta_x}+\frac{\tau_y}{\delta_y})\right]\cos\left(\frac{\tau_x}{\delta_x}\right)\cos\left(\frac{\tau_y}{\delta_y}\right)$
Second order Markov	$\rho[\tau_x,\tau_y]=\exp\left[-4(\frac{\tau_x}{\delta_x}+\frac{\tau_y}{\delta_y})\right]\left(1+\frac{4\tau_x}{\delta_x}\right)\left(1+\frac{4\tau_y}{\delta_y}\right)$

In the present study, the second-order Markovian spatial correlation function is used to get the correlation coefficient between log-elastic modulus values:

$$\rho[\tau_x,\tau_y]=\exp\left[-4(\frac{\tau_x}{\theta_x}+\frac{\tau_y}{\theta_y})\right]\left(1+\frac{4\tau_x}{\theta_x}\right)\left(1+\frac{4\tau_y}{\theta_y}\right) \quad (1)$$

where τ_x and τ_y are the distances between two locations in the horizontal and vertical directions, respectively; θ_x and θ_y are the horizontal and vertical spatial correlation distances, respectively. Two cases of correlation distance include isotropic Θ varied from 0.05 to 20 and anisotropic correlation ratio ε varied from 0.5 to 200 are assumed in the proposed model. Table 2 summarizes the input parameters used during the RFEM simulation.

Table 2. Input varying parameters.

Varying input parameters	Values
COV_E	0.1, 0.2, 0.3, 0.4, and 0.5
Isotropic scale of fluctuation, ($\theta_{\ln(Ex)}=\theta_{\ln(Ey)}$), (m)	0.05, 0.1, 0.5, 1, 5, 10, and 20
Anisotropic scale of fluctuation ratio, ε= $\theta_{\ln(Ex)}/\theta_{\ln(Ey)}$ where $\theta_{\ln(Ey)}$ is constant = 0.1 m	0.5, 1, 5, 10, 15, 50, 100, and 200

The Monte Carlo technique is used to simulate the realizations of the random field (elastic modulus field) and then they passed to the FEM to analyze the elastic settlement (Smith et al., 2013). By repeating this process after a sufficient number of realizations a set of settlement values are determined. These values of the elastic settlements are then statistically assessed. To produce the realizations of the log elastic modulus field 2-D Local Average Subdivision technique is used (Fenton & Vanmarcke, 1990). By using this method each discrete local average given by a realization becomes the average property within each discrete element. So, the elastic modulus that assigned to the ith element can be expressed as Equation (2):

$$E(x_i)=\exp[\mu_{\ln E}+\sigma_{\ln E}G(x_i)] \quad (2)$$

where $G(x_i)$ is the local average over the element centered at x_i of a zero mean.

Fig. (*2*) shows the strip footing rested on the spatially random elastic modulus field in one of the generated realizations. The light regions have the lower values of elastic modulus and the dark zones correspond to the higher values of elastic modulus.

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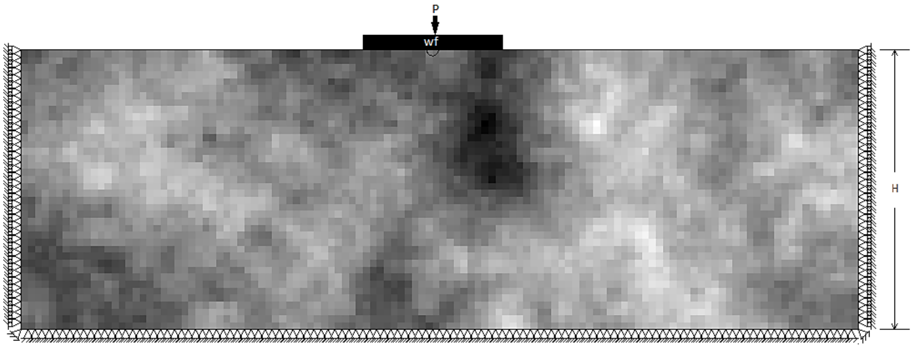

Fig. (2): The finite element mesh with spatial random field of elastic modulus.

3. VERIFICATIONS

In order to be insure about the geometrical size of the model, the effects of boundary side distance on the overall results are first investigated using the developed RFEM. Then, the accuracy of the results obtained by the FEM and also choosing an appropriate number of realizations are verified.

3.1. Boundary side distance and depth of the model

To select a sufficient boundary size of the model, a set of simulation runs of the problem are performed by FEM source code developed by Smith et al. (2013). The mesh is discretized using four nodes quadrilateral elements with 120 elements in horizontal 40 elements in vertical directions (Fig. **(**3). The boundary side distance (η) should be enough to avoid interference with the lateral boundaries, and it is defined as a function of the footing width. To determine the safe boundary side distance several models with different values of η with a constant depth of the soil layer are modeled. The models are analyzed in case of homogenous properties to find out the elastic settlement of the strip footing. The results have shown that the settlement reductions are approximately constant when η > 2.5 as shown in Fig. **(*4***).

Also, the adequacy of depth of the model is investigated by varying the thickness of the layer from 1.5 to 5 m and for different values of COV_E. Generally, the result shows that for small values of COV_E (<0.5) the effects of increasing the depth of the model on the mean settlement values are insignificant. Therefore, and in order to reduce the computational time complexity of the simulations, the depth of the soil layer (H) is set to be a 2.0 m. Fig. **(*5***) shows the effect of the overall depth of the model on the stochastic mean values of settlement for the case with spatial correlation distances $\Theta = \theta_{\ln(Ex)} = \theta_{\ln(Ey)} =$ 0.1m.

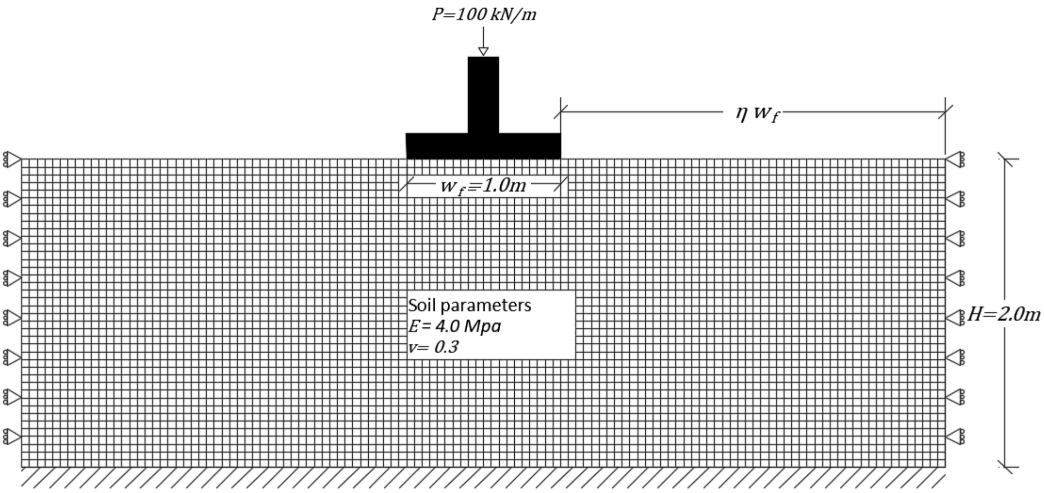

Fig. (3): Finite element model for the soil profile (under deterministic condition).

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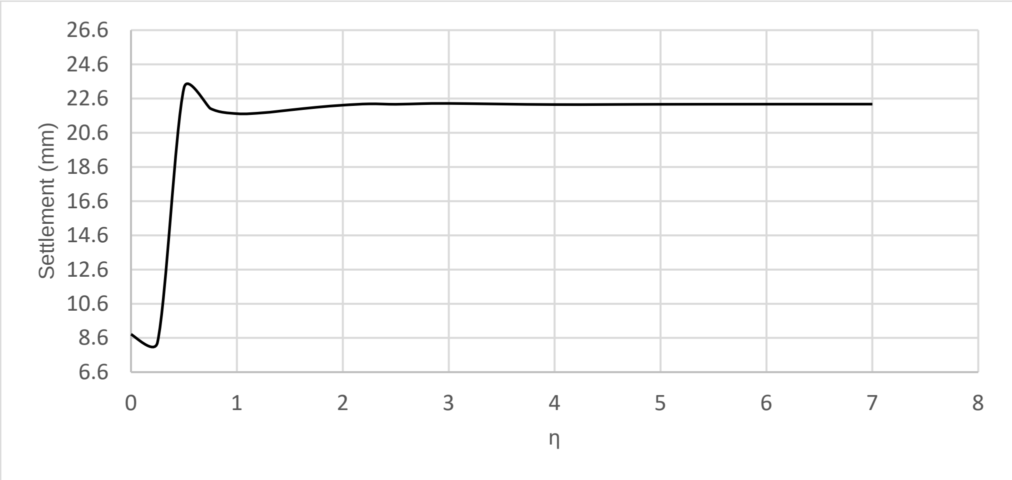

Fig. (4): Side distance effects on the settlement results.

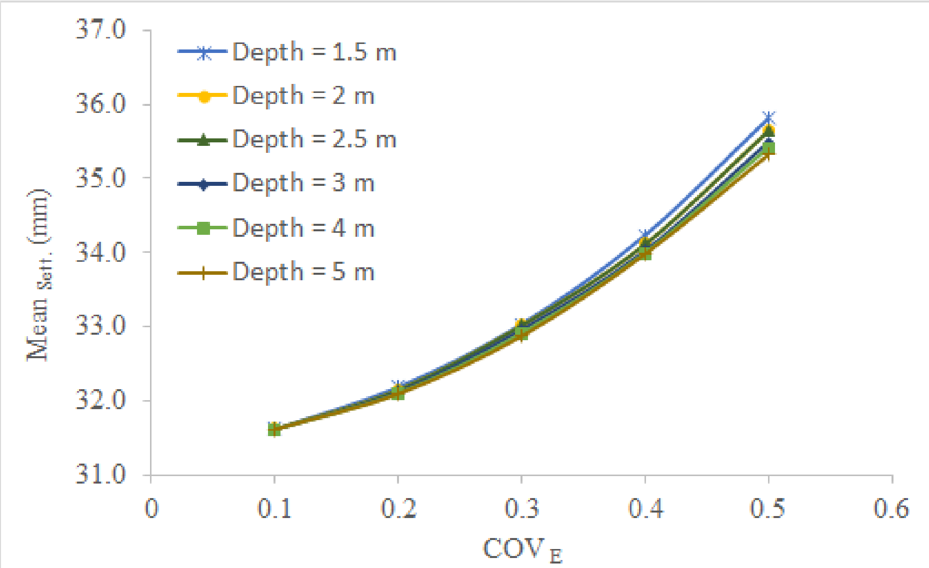

Fig. (5): Output mean settlement vs. coefficient of variation of the random field (COV_E) for different depths of the model.

3.2. Reliability of the FEM analysis

To evaluate the reliability of the FEM calculation result, the traditional approaches using elastic theory are adopted for this study (Equations 3 and 4). Both of these equations are for calculation of settlements at the corner of the flexible footing and the calculated settlement values should then multiplied by a factor of 0.93 to estimate the equivalent values for the rigid footing (Bowles, (1987). Equation 4 is suggested by Poulos & Davis (1974) specifically for the strip footing.

$$S_e = q_o(w_f{}')m\frac{(1-v^2)}{E}I_s \quad (3)$$

$$S_e = \frac{q_o H}{\pi E} m.I_c \quad (4)$$

where S_e is the elastic settlement, q_o is the net applied pressure to the foundation, m is the number of corners contributing to settlement, I_s is the shape factor depends on the width $w_f{}'$, length L of the footing and soil profile depth H. I_c is the influence factor depends on Poisson's ratio and w_f'/H ratio.

In these equations and in order to estimate the maximum settlement at the center of footing, the values of w_f'=0.5m, m =2, I_s =1 are used accordingly. The I_c value equal to 0.77 is used in Equation (4), which is obtained from the

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provided chart by Poulos & Davis (1974). The calculated maximum settlement values from equation (3) is 21.15 mm and from equation (4) is 22.78 mm. The corresponding value calculated by the FEM is 22.35 mm. Therefore, this shows that the numerical result is in a good agreement with the theoretical outcomes. Fig. (***6***) illustrates the final deformed shape of the model under this deterministic condition of the finite element simulation with a 22.35 mm settlement value.

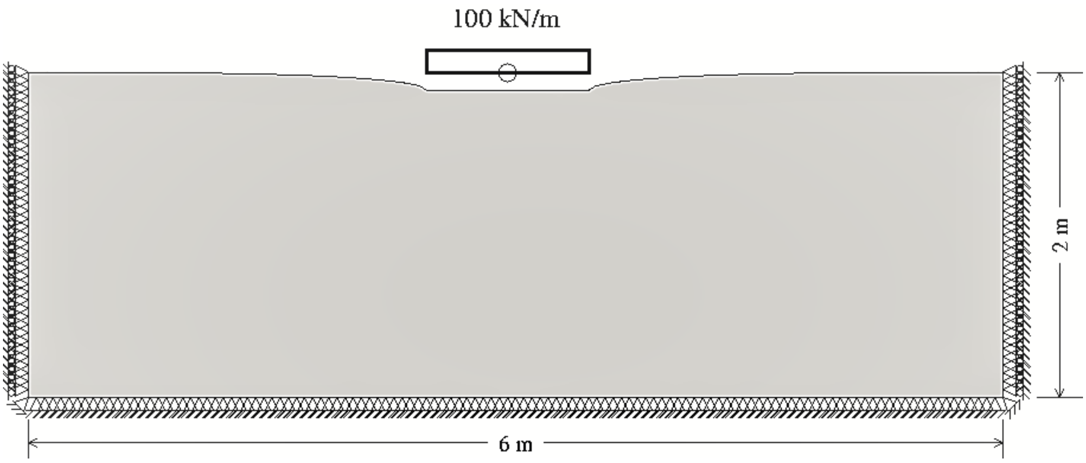

Fig. (6): The final displaced geometry of the model under deterministic condition.

3.3. Sufficient number of realizations

The number of realizations used during RFEM is another important task that should be carefully chosen. Inappropriate selection of this number can effect on the results and also the computational time of the process. To choose an adequate number of realizations (N), the mean values of the settlement results are plotted versus the number of realizations as shown in Fig. (***7***). In this figure, it shows that the mean settlements are nearly to be constant after 1500 realizations. Thus, it can be concluded that above 1500 realizations can be acceptable to use for the current study. Thus, 2000 realizations are employed in this study to each set of the random input parameters.

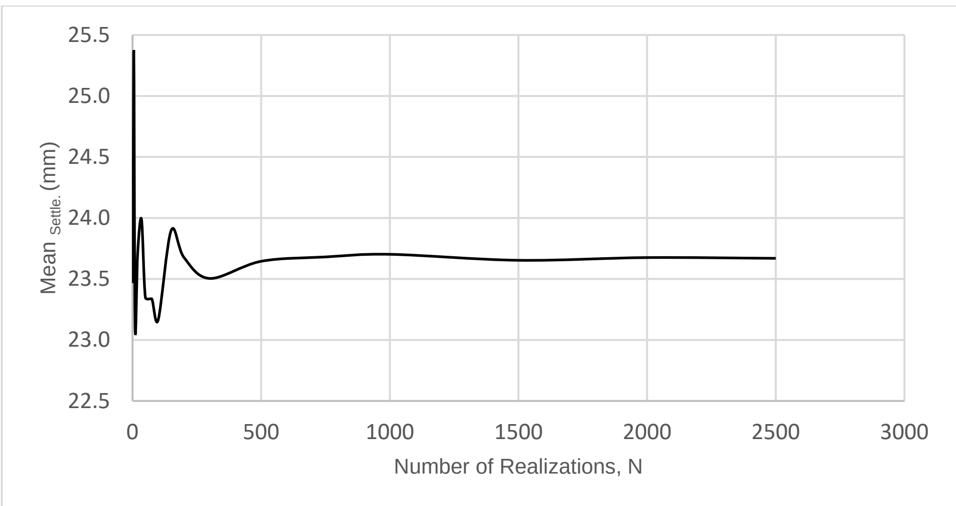

Fig. (7): The effects of the number of realizations on the mean settlement.

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4. RESULTS AND DISCUSSIONS

During the developed RFEM process, the influences of the isotropic and anisotropic correlation distances on the elastic settlement of the rigid strip footing are analyzed. In this process, the various values of the coefficient of variation COV_E are considered. For each of these cases the reported results are based on 2000 different realizations (random fields) of the stiffness as follow:

4.1 Influence of isotropic correlation distance

Fig. *8* and 9 show the influence of the isotropic correlation length (correlation distance in horizontal $\theta_{ln(Ex)}$ and vertical $\theta_{ln(Ey)}$ are equal) and COV_E on the mean (μ_{sett})and the standard deviation (σ_{sett}) of the settlement. When the value of COV_E is equal to 0.1 the value of the mean settlement is approaching the deterministic settlement value (22.35 mm) for all isotropic spatial correlation distances. As the COV_E increases both the mean and the standard deviation of the outputs are increased as well. In general, the associated μ_{sett} and σ_{sett} values by large spatial correlation distances are more than what obtained by small spatial correlation distances. The mean settlement in the extreme condition (COV_E=0.5) increases about 25.9% as compared with deterministic settlement value when $\Theta = \theta_{ln(Ex)} = \theta_{ln(Ey)}$ are equal to 20m. As a result, the conventional deterministic settlement is underestimated. Whereas for the case with Θ= 0.05 m, the mean settlement increasing rate is 12.6%. So, it indicates that the isotropic correlation length and coefficient of variance of elastic modulus directly influence the foundation settlement; consequently, the effect of correlation distance should be considered when the COV_E becomes larger. Fig. **(10**) shows one of the random realizations generated for the cases with Θ = 0.05, 0.1, 1, 1.5, and 20m. In these figures, the darker zones have larger stiffness values.

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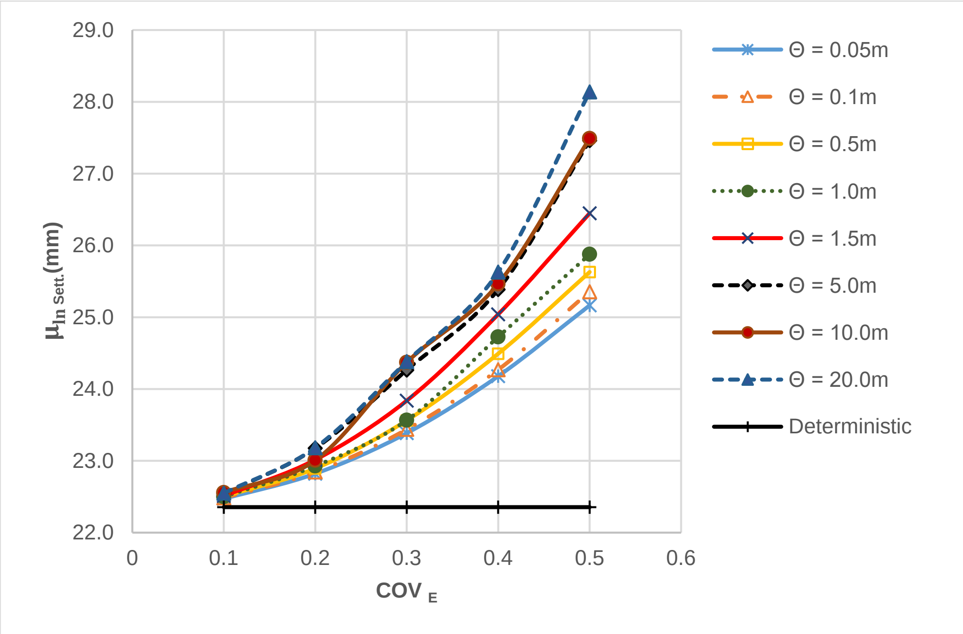

Fig. (8): The relationship between the output mean settlement $\mu_{sett.}$ and coefficient of variation of input (COV_E) for different isotropic correlation distances.

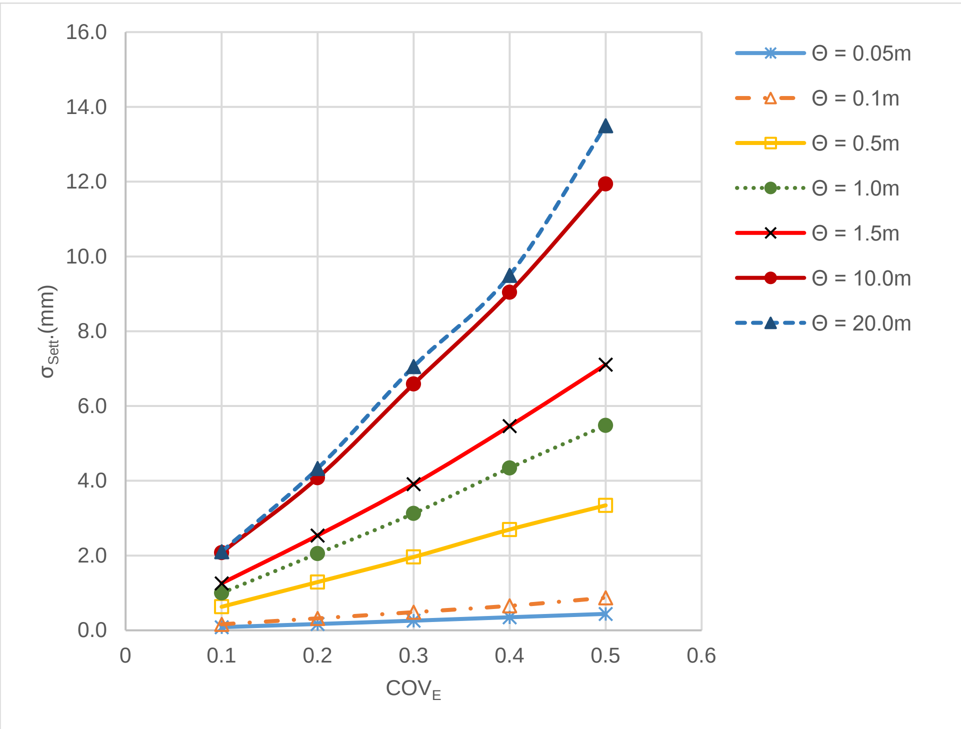

Fig. (9): Variation of standard deviation of output (σ_{sett}) versus coefficient of variation of input (COV_E) for different isotropic correlation distances.

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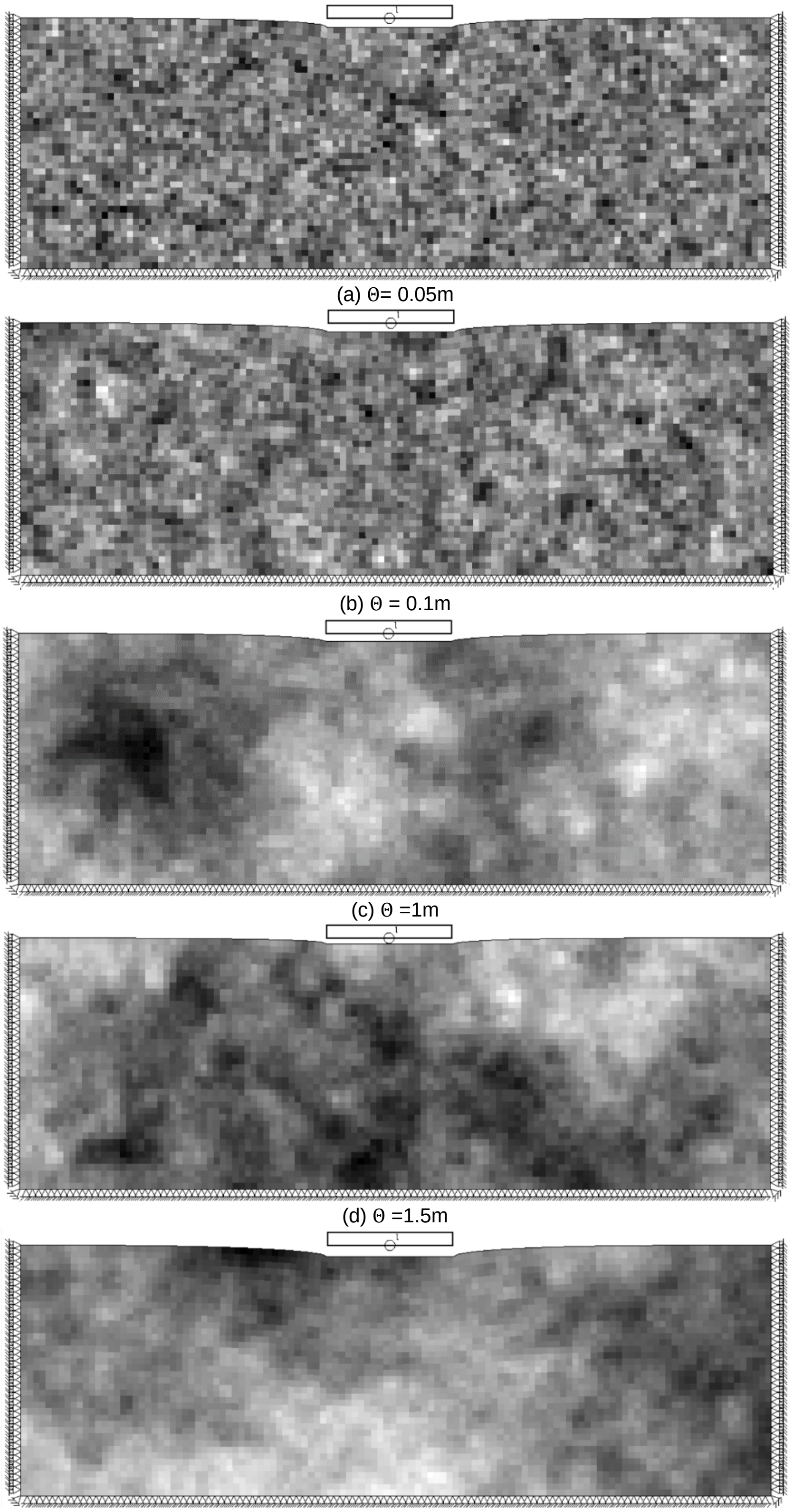

(a) Θ= 0.05m

(b) Θ = 0.1m

(c) Θ =1m

(d) Θ =1.5m

(e) Θ =20m

Fig. (10): Selected realizations of the developed E-fields for different isotropic correlation lengths.

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4.2 Influence of anisotropic correlation ratio

In this phase of the study by varying the horizontal correlation distance $\theta_{\ln(Ex)}$ in the range between 0.05 m and 20 m while keeping the vertical correlation distance $\theta_{\ln(Ey)}$ fixed at 0.1 m in the entire process, the influence of anisotropy ($\theta_{\ln(Ex)} \neq \theta_{\ln(Ey)}$) on the results of the mean settlement are studied. Fig. *11* and 12 depict the relationship between the coefficient of variation of input (COV_E) and mean and standard deviation of the settlement of the rigid strip footing for the cases with different anisotropic correlation ratios ($\varepsilon = \theta_{\ln(Ex)} / \theta_{\ln(Ey)}$). As indicated in these figures both values of μ_{sett} and σ_{sett} are increased with increasing the coefficient of variation for all anisotropic correlation ratios. Also, it is noted that these increasing trends of the results are similar to the cases with isotropic correlation distances (see Figures 8 and 9). The rate of increase of these statistical values of the output (settlement) in the cases with the isotropic correlation distances (especially at COV_E =0.5) is higher than the other simulated under anisotropic conditions. The random field of stiffness in several selected realizations for the cases with ε = 0. 5, 1.0, 2.5, 10, and 200 are presented in Fig. **(13)**.

Huang et al. (2018) argued that the general influence of the vertical correlation distance on the settlement results is more significant than the horizontal correlation length. They recommended considering anisotropic correlation distance to the spatial variability of the soil properties. In order to highlight this impact on the results of the current study, the Fig. (14) is illustrated. It represents the results of variations of μ_{sett} with COV_E for both cases of heterogeneous isotropic and anisotropic soils having the same $\theta_{\ln(Ex)}$ (but different $\theta_{\ln(Ey)}$). The figure clearly manifests that the effects of the vertical correlation length are insignificant for cases with lower values of COV_E and both isotropic and anisotropic random fields almost have similar μ_{sett} values.

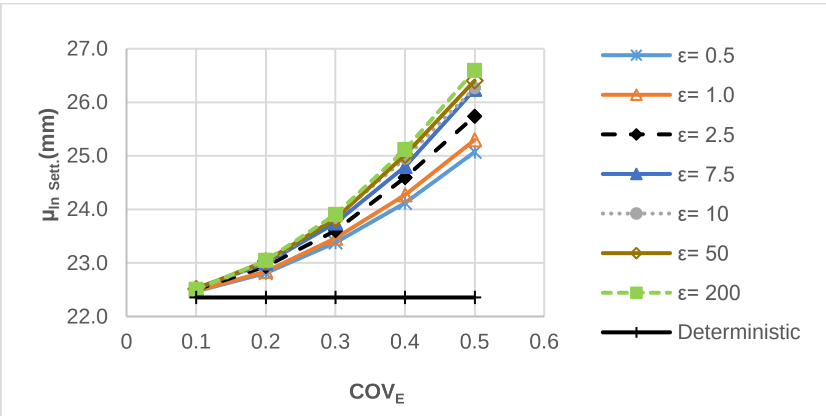

Fig. (11): The relationship between the mean settlement $\mu_{sett.}$ and COV_E for different anisotropic correlation ratios.

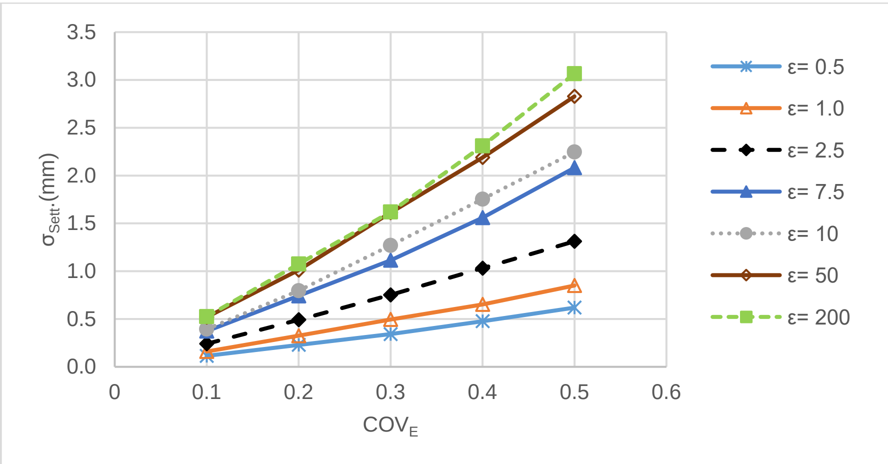

Fig. (12).: Variation of standard deviation of the measured settlement (σ_{sett}) versus coefficient of variation of input (COV_E) for different anisotropic correlation ratios.

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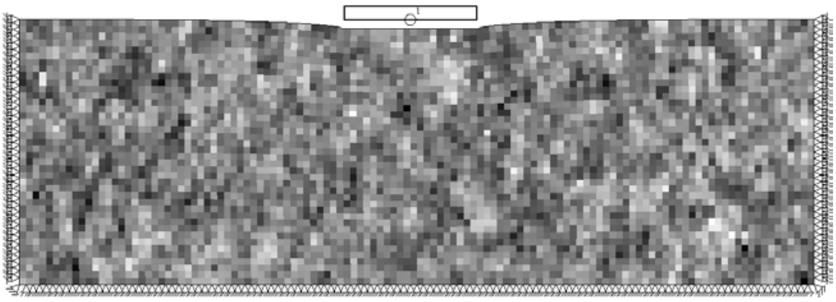
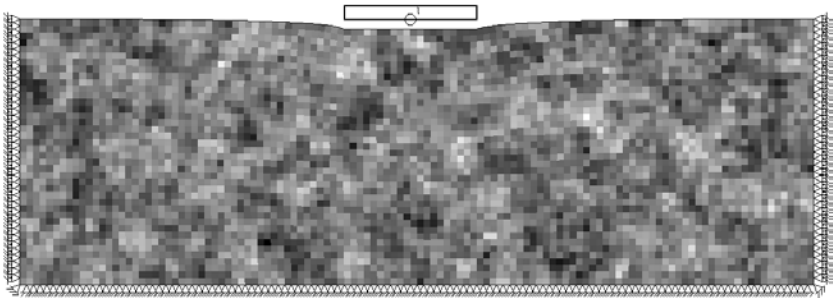
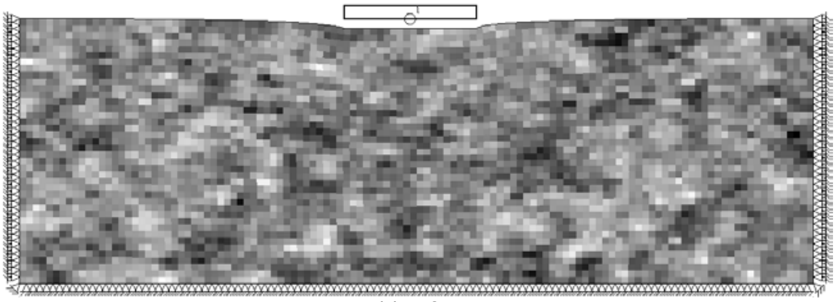


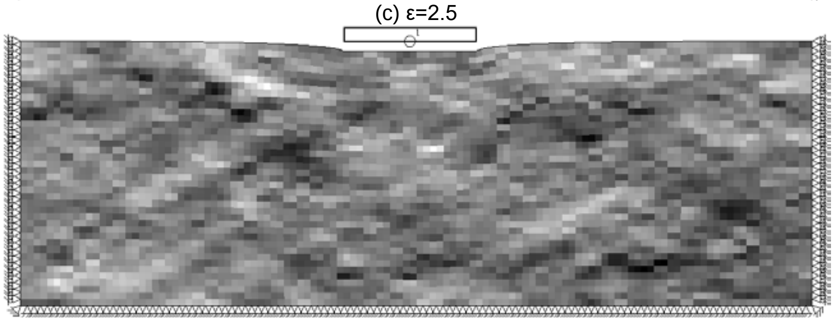
(a) ε=0.5



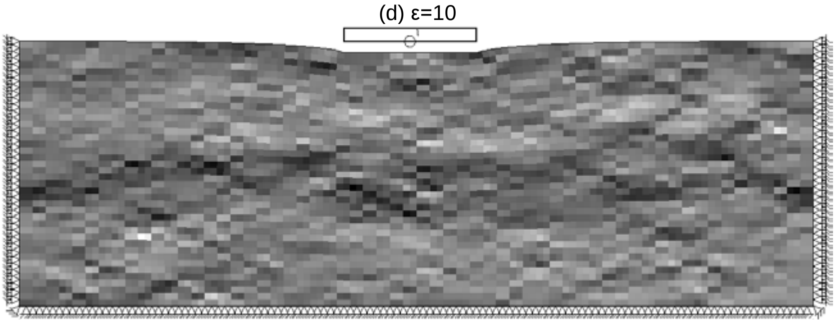
(b) ε=1



(c) ε=2.5



(d) ε=10



(e) ε=200

Fig. (13): Selected realizations of the developed E-fields for different anisotropic ratios.

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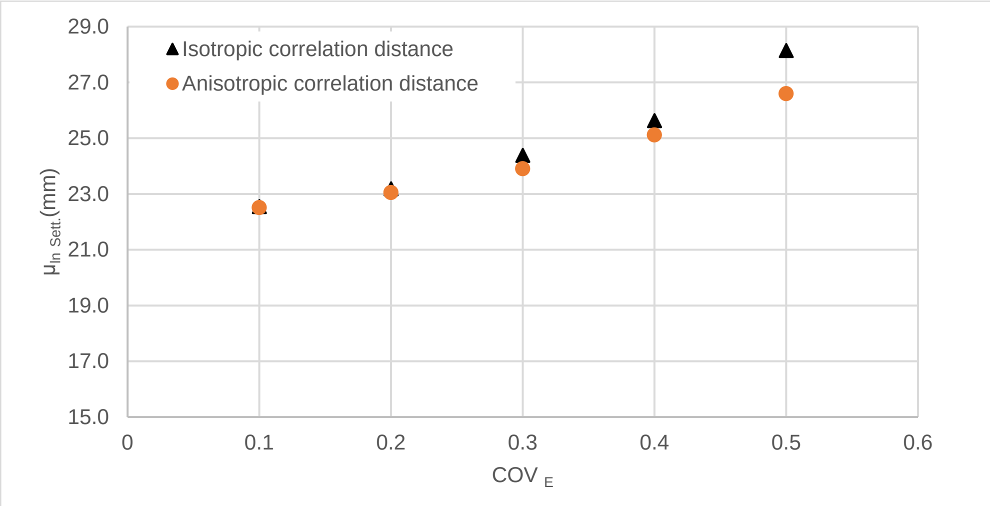

Fig. (14): The influence of the isotropic ($\theta_{lnEx} = \theta_{lnEy} = 20\ m$) and anisotropic ($\theta_{lnEx} = 20\ m, \theta_{lnEy} = 0.1\ m$) correlation distances on the mean of the footing settlement.

5 CONCLUSIONS

Based on the results of the probabilistic settlement analysis of strip footing under the case of isotropic and anisotropic random E-fields the following main concluding points are drawn in this study:

- The results of the footing settlement on the heterogeneous soils are generally larger than the deterministic homogeneous soils.
- The values of the mean and standard deviation of the footing settlement are increased by increasing the coefficient of variation of the input uncertainty (random variable). This increasing trend in the results of isotropic and anisotropic correlation lengths is more pronounced for the case with the coefficient of variation of 0.5.
- In the worst scenario for the cases with the coefficient of variation of 0.5, the large value of isotropic correlation length led to an increase in the mean settlement value by more than 25% as compared with the deterministic settlement calculations.
- The estimated mean and standard deviation of the settlement is increased by increasing the anisotropic ratio. Moreover, these rates of increase for the isotropic random soils are greater than the anisotropic cases (i.e. cases with anisotropic correlation lengths).

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